Persian Two-Level Patterns in Practice.

Tony Lee, February/March 2016

Two-level patterns arise when the spaces between the lines of one pattern (the larger, or upper-level pattern) are filled with tiles of another pattern (the smaller or lower-level pattern). Usually the tiles of the lower-level pattern are so arranged that the pattern is continuous across the lines of the upper-level pattern. That is, in such a case, if the lines of the larger pattern were removed the smaller pattern would present a single, complex pattern covering the whole pattern area. If the upper and lower level patterns are similar, or contain the same shapes (but at two different scales) then such a pattern may be regarded loosely as self-similar. Most discussions of these concepts are concerned with Islamic geometric patterns, where there is frequently a large-scale pattern picked out in various ways — for example, by colour contrast, or greater width of banding — superimposed on a small-scale pattern. Strict self-similarity is rarely achieved in traditonal Islamic patterns.

Recent literature on Islamic patterns has recognised a number of different categories of two-level patterns, but there is one category of tile mosaic in which the manner of execution has been consistently misrepresented in the computer graphic illustrations accompanying a number of modern accounts. In this type of two-level pattern the lines of the large or level-one pattern are widened as bands, separating different areas (polygons, compartments) of this upper level pattern, which are then filled with modular arrangements of parts of the smaller or level-two pattern. Sometimes the bands of the upper level pattern are themselves filled with a lower level pattern, but when the areas between these greatly thickened upper level bands are also filled with a level-two pattern then the two regions, thickened bands and compartments, are still separated by secondary banding. The present essay will thus deal with two types of two-level patterns occurring in present-day Iran.

(1) in this, the most common type, the larger, level-1 pattern is represented as simple widened bands enclosing polygonal areas filled with a smaller, level-2 pattern;

(2) in the second, more elaborate variety, the banding of the level-1 pattern is itself filled with a smaller, level-2 pattern, as well as the areas of level-2 patterns within the compartments of the upper level pattern.

It is a general rule in Islamic geometric patterns that in any enclosed or bounded patterned area those tiles which are on the boundary either meet the boundary at a point, or they have a symmetry axis coinciding with the boundary. That is to say, each edge of the boundary coincides with a mirror axis of the pattern, which may be one of its global mirror axes if the pattern can be extended as a two-dimensional repeating pattern, or merely a local mirror axis. This general rule also applies in two-level patterns, in particular to the compartments of the level-1 pattern which are filled by the level-2 pattern.

In the two categories just defined, where the lines of the larger pattern become substantially widened bands, the compartments occupied by the smaller, level-2 pattern necessarily become reduce in size, and especially, their edges become shorter than the original lines of the upper pattern. The important point to note, however, is that the edges of adjacent compartments are not necessarily equally shortened, and even more to the point, the patterns, and therefore the tiles within those adjacent compartments become reduced to different sizes. This is an inevitable consequence of the transformations undergone by the differential size reduction following the widening of the lines of the upper level pattern as thickened bands.

A pattern commonly used for the upper level of a two-level pattern is the pattern of 10-stars shown in fig. 1. On the left is the pattern on its own, and on the right its individual polygons are filled with a level-2 pattern, shown in red. Since the lines of the level-1 pattern are not thickened, they could be removed to leave a complex decagonal pattern continuously covering the rectangle shown, from corner to corner. All the polygons of the level-1 pattern are therefore filled with elements of the level-2 pattern at one and the same scale.

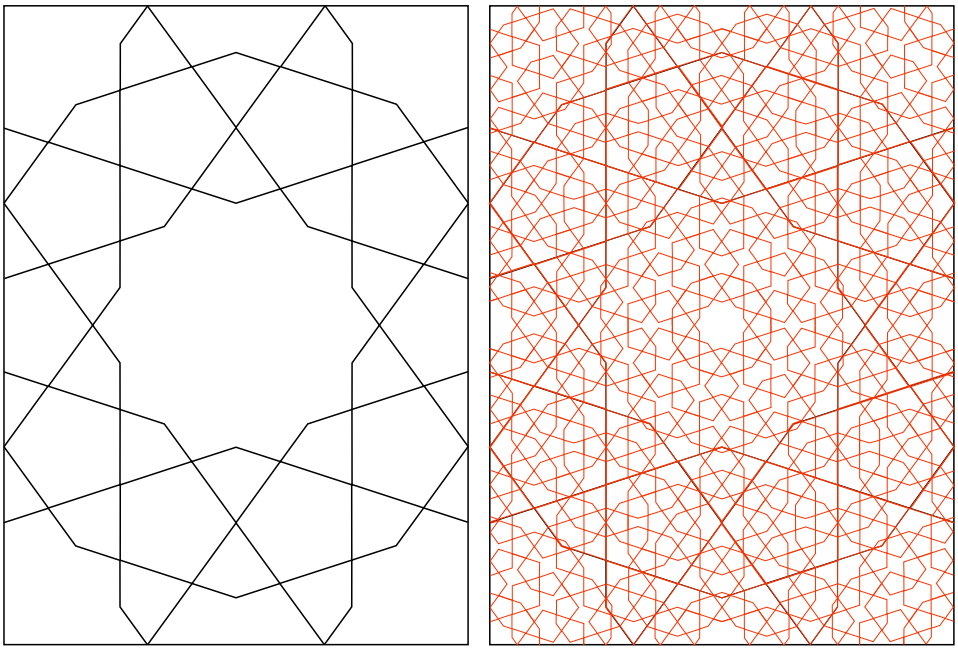

Fig. 1: A common decagonal pattern (left), and as part of a two-level pattern (right).

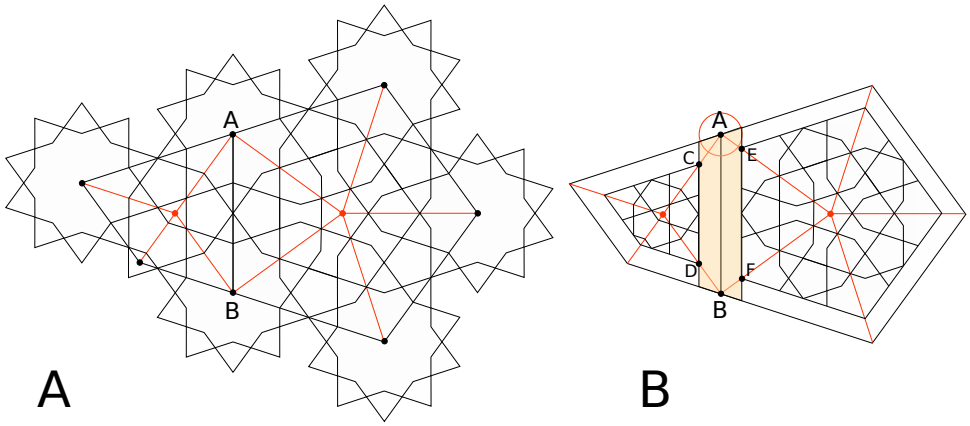

Fig. 2: On the left are two filled polygons from the two-level pattern in fig. 1, sharing a common edge AB. On the right, the line AB has been replaced by a widened band (shaded), and the original shared edge has been split into CD and EF, the reduced edges of the kite and the pentagon, respectively.

If we take an adjacent kite and pentagon from the two-level pattern in fig. 1, they share a common edge AB, as in fig. 2A. If the lines of the level-1 pattern are replaced by widened bands, then the contents of the original kite and pentagon become transformed as in fig. 2B. The initial shared edge AB now becomes CD in the kite, and EF in the pentagon. But note that CD is no longer equal to EF, and more importantly, the scale of the pattern in the transformed

kite is now a good deal smaller than that of the pattern in the pentagon. This is a general feature of unequal filled polygons in a two-level pattern following the introduction of widened banding in the level-1 pattern. Note that in the case of kite and pentagon, because the bisectors of their internal angles all meet at a single point (red dots in fig. 2) they retain their original shape after shrinking, and the transformed vertices in each case move along the angle bisectors towards the centre of each polygon. The thickness of the band in fig. 2B is purposely exaggerated, to emphasise the difference in scale resulting from banding, but the scale difference will still be present, whatever the thickness of the banding.

If the internal angle bisectors of a polygon do not meet at a single point, then the polygon will not retain its exact shape after the presence of level-1 banding produces shrinking. This is shown clearly by the "twinned-pentagon" shape in fig. 3. Here the outer edges represent the original positions of lines in the level-1 pattern (half the shape is visible midway along each border of the left-hand pattern in fig. 1). Equal thicknesses of banding on each edge reduces its original size, shown successively by the smaller polygons within the outer edge, but the initial shape gradually changes, until the outline in red is equivalent to a pair of small pentagons back to back.

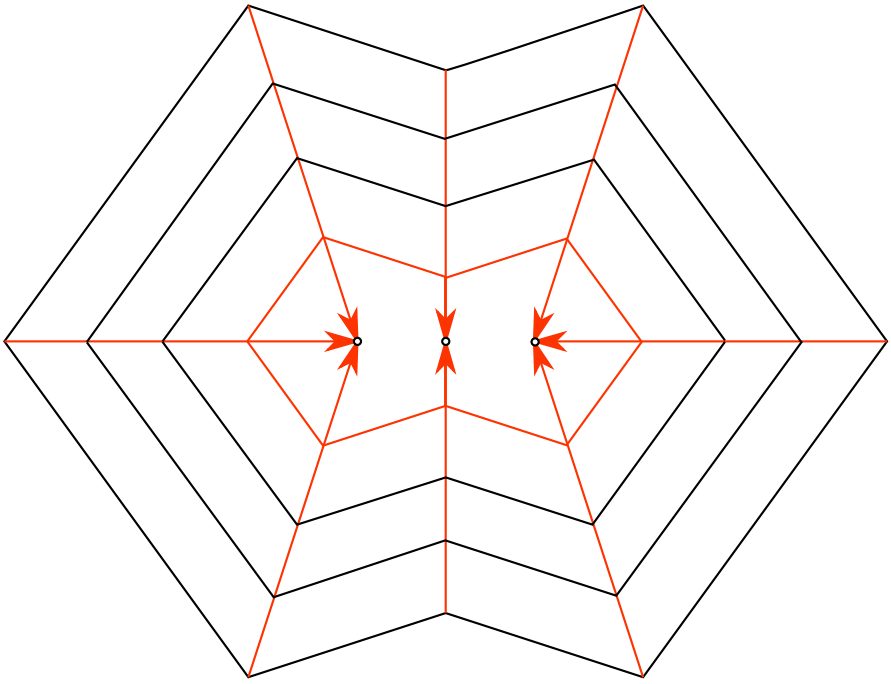

Fig. 3: Reduction in size of a twinned-pentagon shape by successively thicker banding.

The changing shape of polygons such as the twinned-pentagon may cause difficulties in tile layout in two-level patterns if the banding becomes too wide. Luckily most traditional examples seem to restrict the width of banding to fairly narrow strips, so difficulties caused by changes in shape will be minimalised.

The effect of banding on the relative scales of level-1 compartments each side of an edge of the upper level pattern can be shown in a close-up photograph, as in figs. 4 and 5. These images are details of an example of the second category of two-level patterns mentioned above, in which the banding in the upper level pattern is filled with a lower level pattern, theoretically the same scale as the level-2 pattern in the upper level compartments — for example, the kite

and the pentagon. In this case there occurs secondary banding (pale blue lines) between the banding pattern (white and blue tiles) and the compartments pattern (yellow and black tiles).

Fig. 4 : Detail of a two-level decagonal pattern from Isfahan, from the Madar-i Shah Madrasa. The primary banding has been filled with its own level 2 pattern, while secondary banding appears between primary banding and the level 1 polygons. © R. Henry.

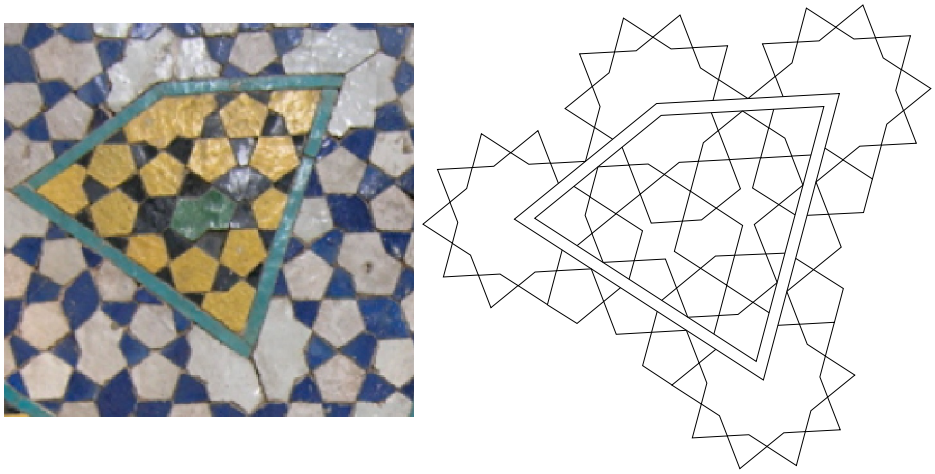

Fig. 5 : Detail of the kite from fig. 5, with a diagram of its geometry, and showing the difference in scale between kite and surround.

Fig. 5 shows the region of the kite in greater detail, together with a geometrical drawing illustrating the difference in scale between level-2 patterns each side of the secondary banding. It is clear from both the photograph and the drawing that the star-to-star distance is shorter along the edge of the kite than in the pattern surrounding it. This follows from the fact that the 10-stars are centred on the corners of the various areas of the level-2 pattern, but because of the presence of the banding the centres of the 10-stars are at slightly different locations on both sides of the banding. Thus, the two edges of each segment of banding still lie along local mirror axes of the patterned area adjacent to them.

There is one further point which needs to be mentioned concerning the presence of secondary banding — that is, the banding which occurs between the level two patterns occupying the area of the level-1 banding and the level-1 compartments. The secondary banding cannot be widened both sides of its theoretical axis, since the filled level-1 banding, being continuous throughout the pattern as a whole, cannot shrink relative to the original size of its borders. Instead, secondary banding must be widened on one side only, that facing the interiors of the enclosed compartments which are filled with the level-2 pattern. Secondary banding, therefore, is best regarded as part of the compartment it surrounds, so secondary banding and compartment should perhaps be drawn as a single module.

Practical application of variable scaling.

The slight difference in scale between level-two pattern inside and outside the kite is clearly seen on close examination of the image on the left side of fig. 5. The 10-stars are obviously of different sizes, and the twinned-pentagon (green) inside the kite is clearly smaller than those (white) outside. The vase-shaped regions (black and blue respectively) are also clearly of different sizes, as are the tiny kite-shaped tiles. Some such adjustments must obviously be carried by the artisans involved in assembling tile mosaics of this type, since the difference in scale forced by the presence of banding cannot be avoided.

The usual computer graphic imitations of two-level patterns start by drawing the lower level pattern as a continuous whole, at a single scale, then simply overlaying the upper level pattern, often with slightly thicker lines, along mirror axes of the lower pattern. Inevitably, if the upper pattern lines are thickened, they must obscure parts of the lower level pattern, specifically, immediately each side of the mirror axis they cover. This is not, however, the manner of execution of traditional two-level tile mosaics, in which the lower level patterns within each modular compartment retain their bordering local mirror axes, which still abut against the edges of the banding tiles of the upper level pattern. In practice, the 10-stars centred on the nodes of the level-1 pattern no longer have their regular, circular appearance in traditonal two-level mosaics, in contrast to computer graphic representations in which they appear perfectly regular.

It is uncertain whether the unavoidable scale changes in the various modules of the lower level pattern affect the design and layout of the final tile mosaic. Previous analyses of decagonal two-level patterns claim that they were composed with the aid of "girih tiles" or various series of underlying polygons. Such methods can assist the layout of a lower level-2 pattern at a constant scale, but do not take into account the additional dimensions necessitated by the presence of widened banding in the upper level pattern, nor do they take into account the varying scales of the modular compartments of the level-2 pattern.

The following figures give a few examples of two-level decagonal patterns, taken from traditional mosaics in Iran, in which an effort has been made to imitate the methods of assembly actually used. In particular, the differences in scale necessitated by the presence of banding. This is easy to do in scalable vector graphics software, where a grouped set of shapes can be enlarged or diminished at will to any desired size, then duplicated and rotated or dragged into position as required. The drawings are intended to illustrate the principles involved, but I have not necessarily copied every idiosyncratic placing of individual tiles observed in the original tile mosaics.

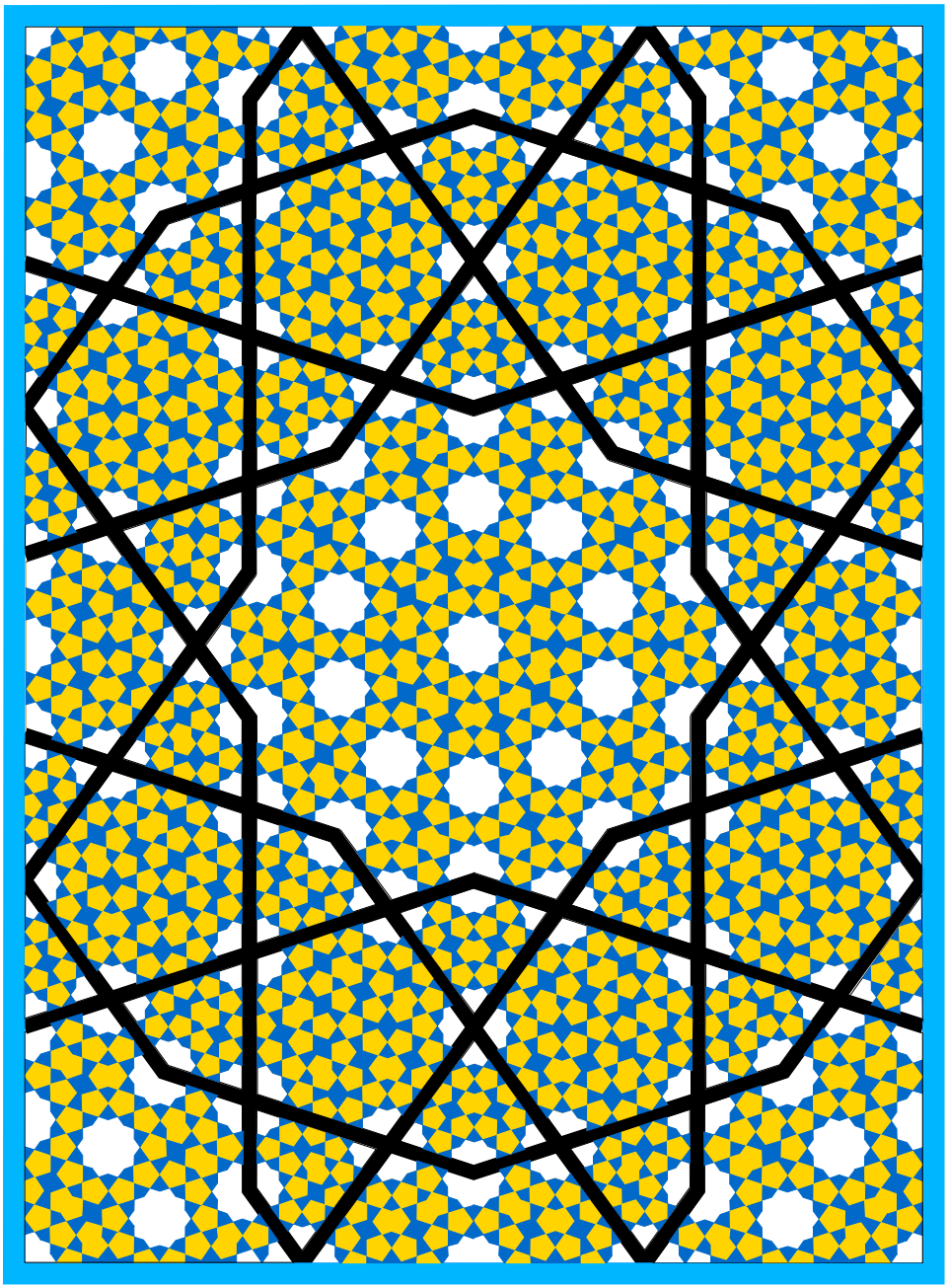

Fig. 6: Two-level tile mosaic from arch spandrels in the Darb-i Imam shrine at Isfahan (pattern completed and rotated 90°). Note that the black banding does not overlap the edges of the individual lower level tiled areas, which would have been the case if the level-2 background had been drawn at a constant scale, as a continuous background pattern.

An image of the original mosaic can be seen in Wade IRA 0837, which shows slight inaccuracies in the layout of the black banding of the upper level pattern. The orientation of the patterns of tiles inside the pentagonal modules is variable in the original, but these are not accurately copied in the drawing above. The misshapen appearance of those white 10-stars which are crossed by upper level banding is a feature of the traditional original, reflecting the manner of assembly of the original mosaic.

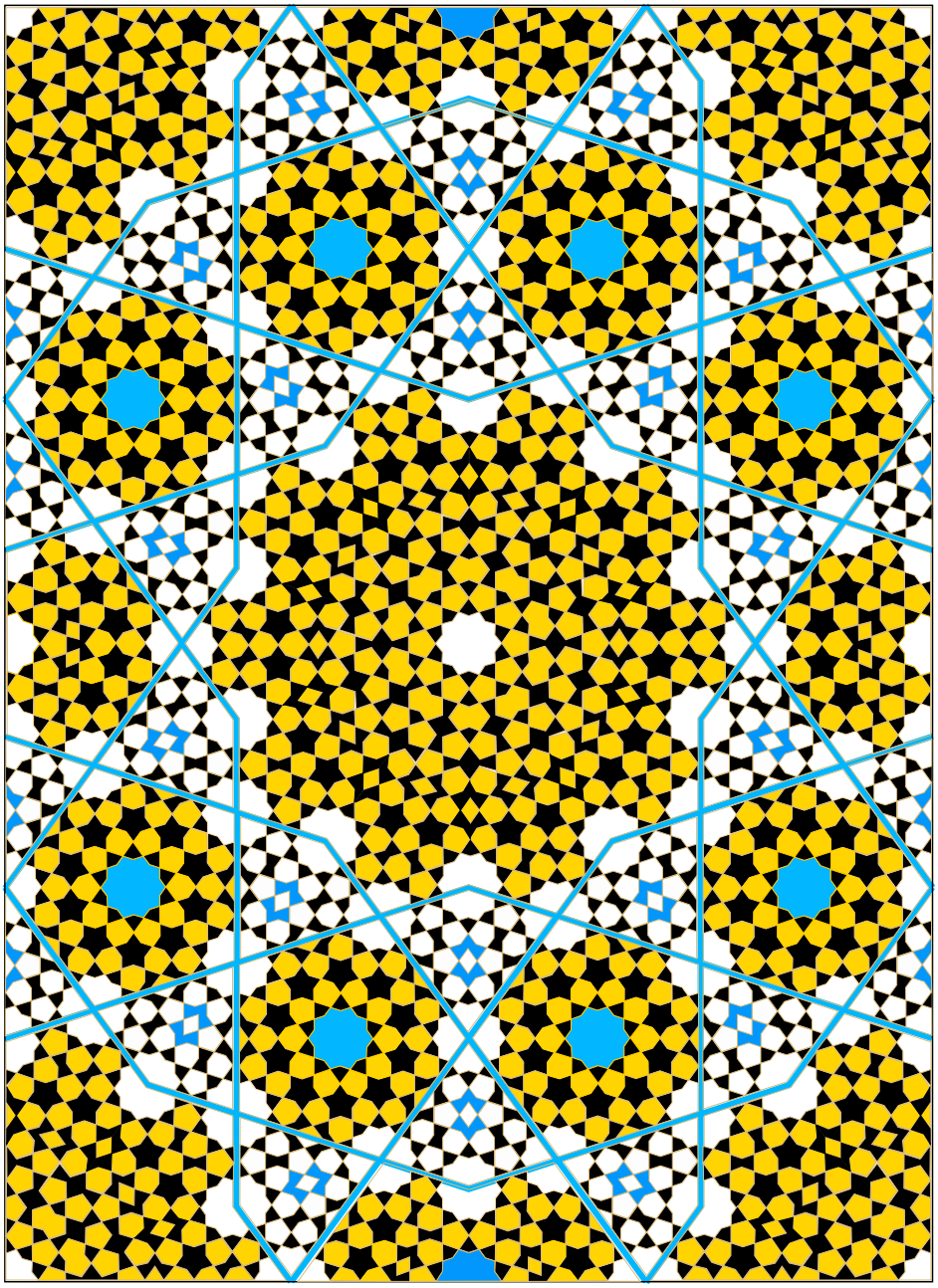

Fig. 7 : A slight variation on the previous pattern. Metrically, the relation between level 1 and level-2 patterns is the same in figs 6 and 7. Scale changes are far less noticeable in fig. 7 because of the much narrower level-1 banding. This pattern is from the Madar-i Shah madrasa in Isfahan.

Figures 6 and 7 show simple level-1 banding, slightly thicker in the Darb-i Imam example, which therefore displays a more noticeable scale change between kite, pentagon and 10-star modules.

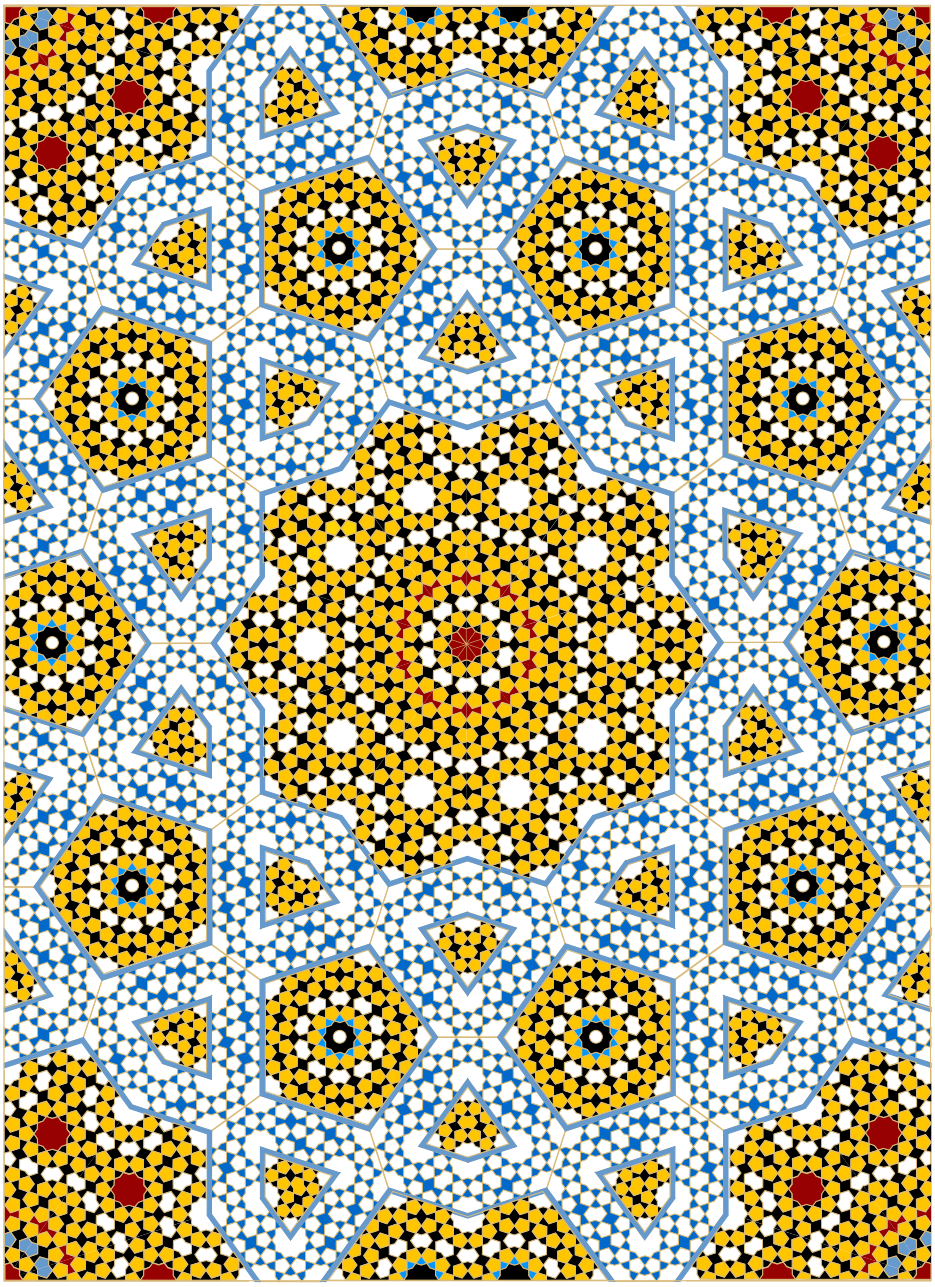

Fig. 8: Based on a much larger panel at the Madar-i Shah madrasa in Isfahan. The panel above uses widened banding from the pattern in fig. 1 as its upper level pattern, but here the banding is filled with part of the lower level pattern. Note that the banding pattern (white and blue tiles) is continuous right across the panel, but that the lower level patterns in the compartment modules (mainly yellow and black tiles) are discrete and at different scales from each other and from the banding pattern. The original pattern is rotated 90° relative to the drawing above. I have slightly modified the distribution of colours in my graphic compared to the original. The banding pattern in the computer graphic is easy to assemble if we regard it as repetitions of kite-shaped (or half-kite-shaped) modules. I have retained the dividing lines between modules (faint brown lines) to make this clear. The pattern here is similar to the mosaic from which fig. 4 has been taken. See Wade IRA 1118.

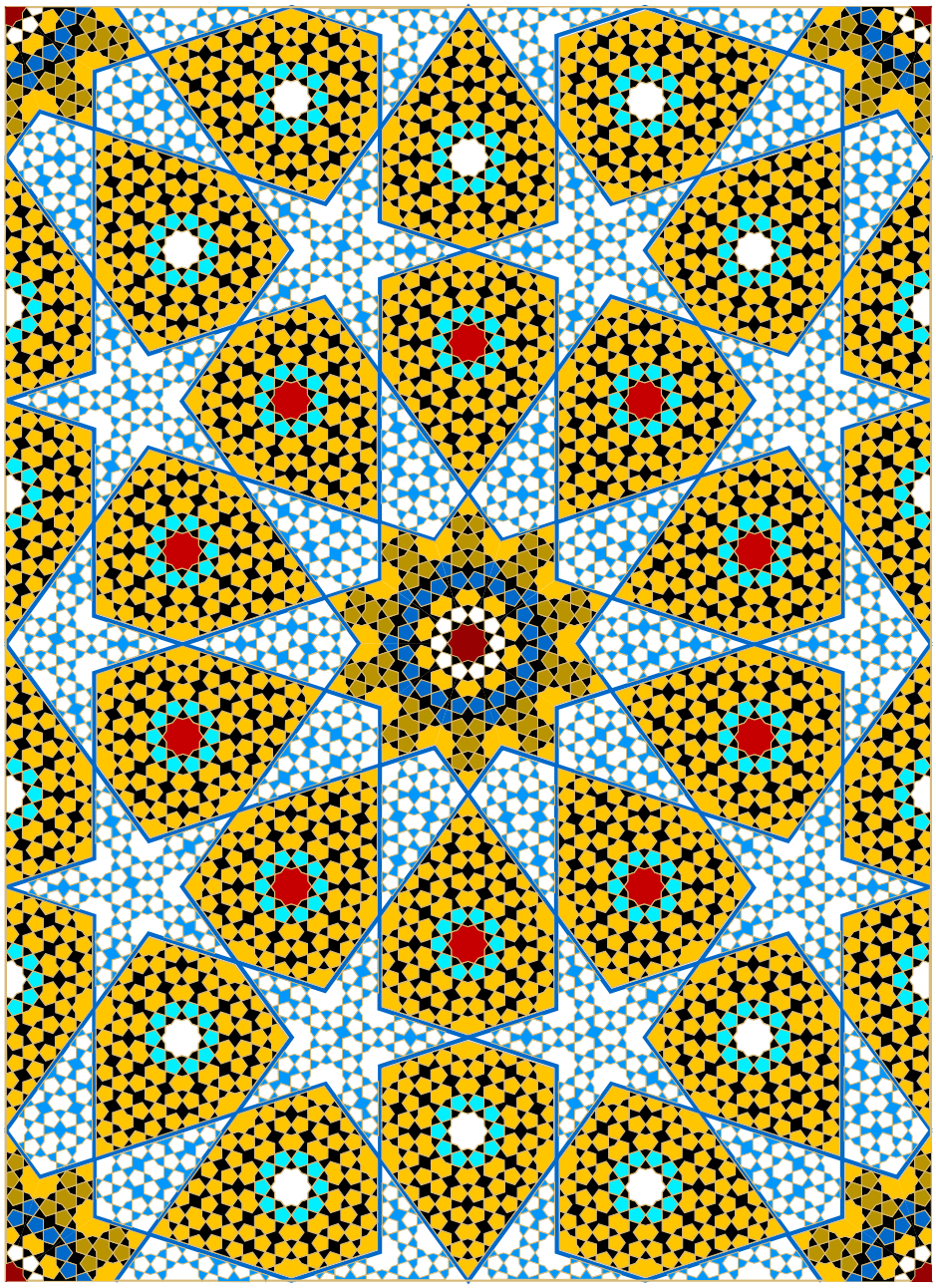

Fig. 9: A companion mosaic to that of fig. 8 from the Madar-i Shah madrasa in Isfahan. I have rendered some of the colours a little imaginatively, but on the whole the graphic above follows the original fairly closely. This is the rosette version of the star pattern shown in fig. 1, and in the upper level patterns of figs. 6, 7 and 8. Note that the sequence of decagonal tiles is noticeably out of synchronization across the blue edge separating the 5-stars from the petals of the 10-fold rosettes. This is also a feature of the original mosaic and is confirmation of the methods of layout advocated in this essay.

Again, as in the case of the mosaic depicted in fig. 8, I have not attempted to copy the slight differences in the precise orientation of tiles in the centres of the various 5-stars and kites. An image of this mosaic can be seen in Wade IRA 1116.

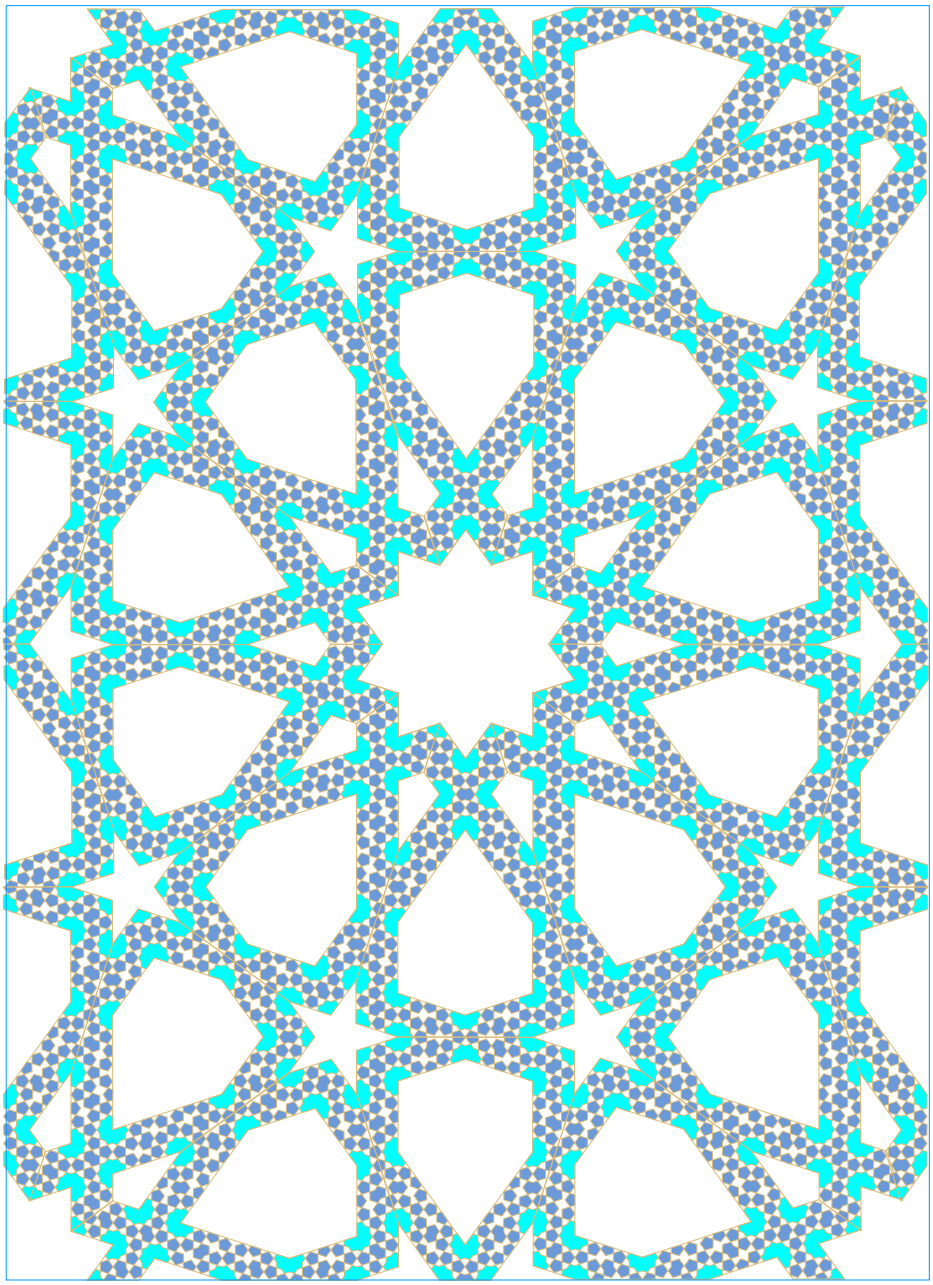

Fig. 10: The widened lines of the upper level pattern from fig. 9 are here filled with a level-2 pattern which is the same as a two-level design in the Topkapı Scroll (Cromwell 2010 fig. 14). However, a completely filled level-2 pattern would not be possible, mainly because the stars centred on the reentrant angles of the pentagrams would overlap one another. A slightly more complex two-level pattern covering the spandrels of an arch in the Masjid-i Seyyed in Isfahan overcomes this difficulty - see fig. 11 - but this composition has difficulties of its own.

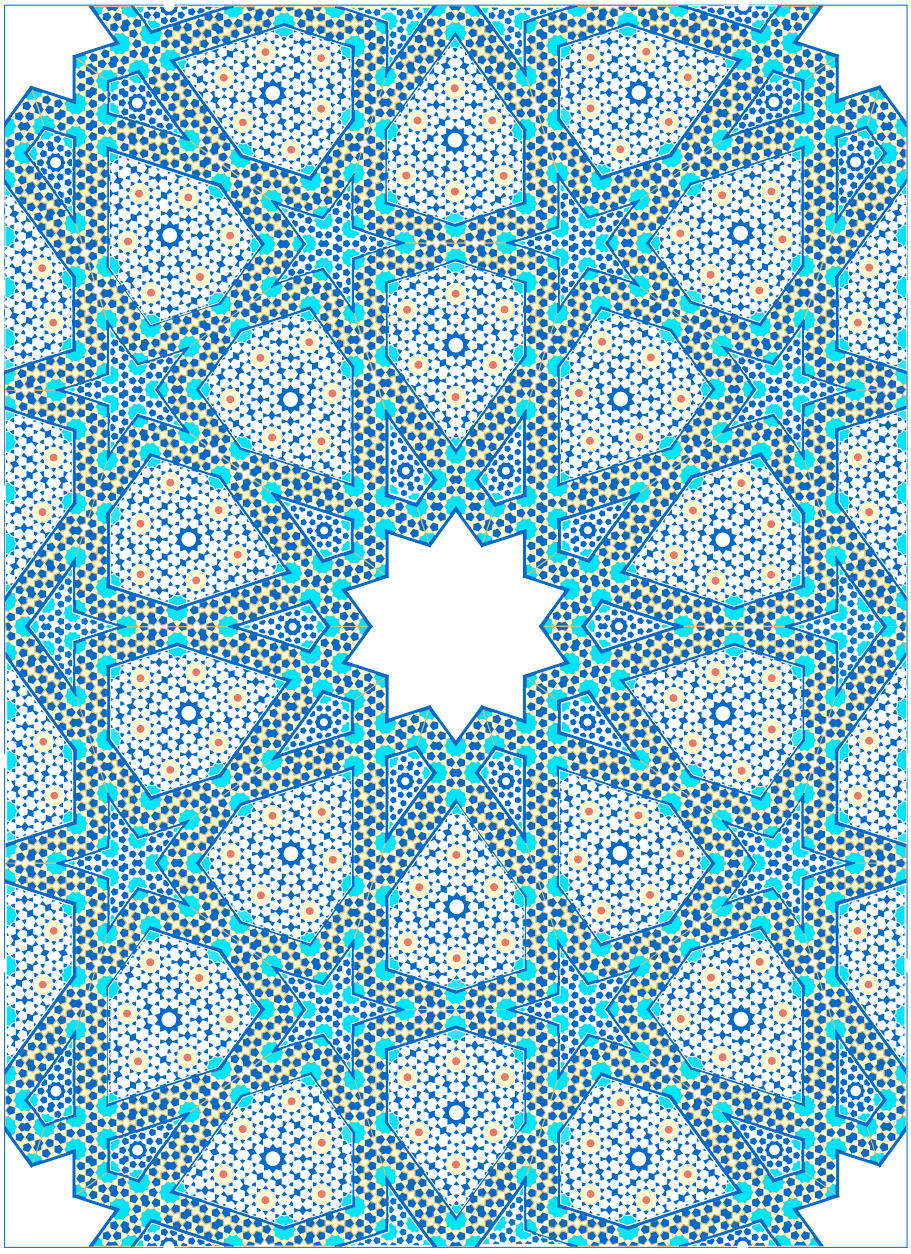

Fig. 11: A two-level design based on a mosaic on spandrels over an arch at the Masjid-i Seyyed in Isfahan. The pattern here is rotated 90° compared to the original. The level-2 design in the central star has been omitted, since the original mosaic does not contain any consistent or symmetric arrangement of tiles in this region.

The original mosaic at the Masjid-i Seyyed, from which fig. 11 has been taken, has very thin strips of secondary banding between the level-2 pattern in the bands of the upper level pattern, and that in the various compartments between the bands of the upper pattern. However, because of the very acute angles of the small kites, peripheral 5-stars and darts, the level-2 pattern inside these is very obviously at a much smaller scale than that in the upper level banding.

As initially laid out for fig. 11, these separate areas of level-2 patterns were drawn at the same scale, but shrinking them to fit the smaller spaces remaining after secondary banding automatically leads to a reduction in scale, as is also necessarily the case in the original mosaic.

The acute sectors of the pale blue 10-stars occupying the acute angles of the peripheral 5-stars and the small kites in most cases appear to have been squeezed out of the rest of the 10-star, although in some cases the mosaicist seems to have taken pains to reduce this effect. The most obvious evidence of the reduction in scale in certain areas of the level-2 pattern is in a comparison of the sizes of the pentagonal tiles each side of the secondary banding. This is very clearly shown on magnification of the drawing in fig. 11, but it is also quite obvious on the original mosaic (I am extremely grateful to David Wade for providing me with a high-resolution image of this mosaic).

Theoretically in a two-level mosaic of this type the location of the secondary banding initially acts as a local mirror axis to the pattern on both sides. After the introduction of secondary banding has led to scale reduction most traditional examples still attempt a kind of "topological" mirroring, in that the same sequences of shapes are retained each side of the secondary banding. However, if we attempt to complete a pattern inside the central 10-star on this basis, we discover that it becomes extremely difficult if not impossible to arrange the usual set of decagonal tiles in a 10-fold pattern. Nowhere in the original mosaic have the artists managed to produce an arrangement of standard "decagonal" tiles which could be completed as a 10-fold design for the central star, and this fact probably reflects the extreme difficulty, and perhaps even the impossibility of doing so (areas *a*, *b* and *c* on fig. 12). This appears to have been something of an embarrassment to the original artists, since they were obviously struggling to fill these areas with what seem almost random arrangements, with little or no symmetry. For this reason, I have left the central 10-star empty, rather than attempting to invent an arrangement of my own.

Overall, the Seyyed Mosque spandrel pattern is a superbly conceived composition, achieved in a masterly manner, but it does have at least one mistake - on the left hand spandrel part of the upper pattern banding was evidently allowed to become slightly distorted, with the result that the area of an adjacent petal was a little too large (area *p* on fig. 12). Consequently the larger area had to be filled by adding an extra row of tiles along two of its edges. This seems to confirm that the level-1 banding was laid out first, then the various compartments filled in afterwards. Fig. 12 shows that the spandrel pattern, if extended as a two-dimensional repeating pattern, would be slightly different to the one in fig. 11. A low-resolution image of the whole two-level pattern may be seen in Wade IRA 1526.

It seems to have been a general principle in two-level patterns, in which the lines of the larger, or upper-level pattern were thickened as substantial bands, that any lower-level pattern drawn in the polygons or compartments of the large pattern should treat the edges of the thickened bands as its boundary, with the inevitable result that the isolated areas of level-2 pattern have to be drawn at different scales, depending on the chosen thickness of the upper level banding, than would have been the case if the compartments had met one another along a geometrical line. This will always be so, if a discrete patterned area is separated from an adjacent, similarly patterned area by an appreciable thickness of either primary or secondary banding, and is a geometrical property, as demonstrated in figs. 2 and 5 above. The fact that lower-level patterns have this relation to the thickened banding of an upper-level pattern is widely appreciated, but the fact of the necessary change of scale does not seem to have been mentioned, although it is perfectly visible on traditional two-level mosaics, as seen in fig. 5.

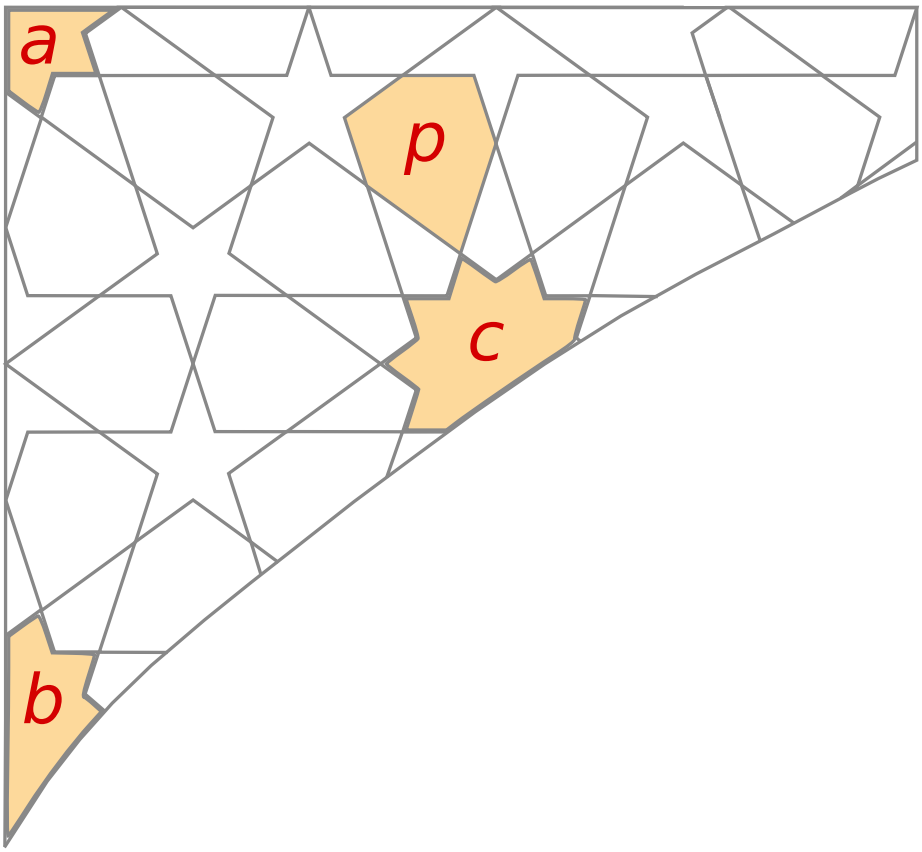

Fig. 12: The scheme of the upper-level pattern on the left spandrel of the Masjid-i Seyyed. The locations of the central stars *a*, *b* and *c* are marked. Area *p* is the petal which had to be inadvertently expanded slightly, because of a misplaced level-1 band to its right.

Selected papers dealing with two-level patterns.

(1) Bonner, Jay, 'Three Traditions of Self-Similarity in Fourteenth and Fifteenth Century Islamic Geometric Ornament'. Proceedings ISAMA/Bridges 2003.

(2) Lu, Peter J. and Paul J. Steinhardt, 'Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture'. Science 315:1106-1110, 2007.

(3) Cromwell, Peter R., 'Cognitive biases in Islamic 5-fold Ornament'. Mathematical Intelligencer 31(1): 36-56, 2009.

(4) Cromwell, Peter R. 'Islamic Geometric Designs from the Topkapi Scroll II: A Modular Design System'. J. Maths. & the Arts **4** (2010) 119-136.

(5) Cromwell, Peter R., 'Modularity and Hierarchy in Persian Geometric Ornament'. Nexus Network Journal (2016), pp. 1-48.