Manual for the Tiling Database

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1 Introduction

This site consists of a data base and search facilities to locate items in the collection. Unlike most data bases, the items are images, but ones that have substantial structure. This structure is recorded in the data base making searching and retrieval possible.

This manual describes how the system works from the point of view of the user. It does not give implementation details.

2 Scope

The system only covers geometric patterns. Moreover, the pattern must consist of tiles which are polygons. Hence the edges are straight lines. The patterns must be on a flat surface. (A small number of images are provided which break these conditions just to complete some well-known collections.)

The other major restriction is that the tiles must be fitted together by means of local connection rules, that is, every edge has a matching edge. This does allow tilings which have a centre and hence are not repeat patterns.

The main emphasis in this collection is on classical Islamic patterns, since it seems possible to record almost all of these.

3 Overview

A simple means of viewing the nature of this collection is to use the facility to generate twenty random patterns, click here. The small version of each pattern can be selected to give the HTML page which gives further details of that pattern.

Another simple method of understanding more about the system is to look at the Statistics page. Numerous lines contain links to parts of the system which may be of interest.

4 Structure of the System

4.1 Sources

Currently, almost all of the material recorded here has been extracted from publications. Hence for every item, there is a list of publications that refers to it. The complete list of publications is here: HTML. Each publication is referenced by text string enclosed in square brackets. In addition, some patterns are taken from David Wade's web site, in which case, a link to the source photograph is given. Links are also provided to the V&A collection.

In those cases in which the author has been provided with information that has not been published or placed on the Internet, the 'publication' [pc] is used (standing for personal communication).

Those publications that have a Pressmark are ones which have been located via the National Art Library catalogue. This catalogue can produce details in Unicode (UTF-8), which is used here.

If you want an ordinary publication containing much of the information here, you can purchase on-line the CD/booklet: [3] or the book: [6].

Given an actual source, transcribing this into a high-quality digital image is not straightforward. Many older artefacts are in poor condition, or sometimes the actual photograph does not have the resolution needed to see the detail. The majority of the source material is not in colour so that the colours used here are often an invention.

Over the next few years, an increasing number of older publications will be available on the Internet. Many academic papers are available that way, at least from the author. However, Internet references (URLs) often vary and hence this system does not store the URL. Instead, those publications available on the Internet are marked as such. As far as possible, a check is made that the current URL can be found using a search engine.

4.2   Producing the digital images

The majority of the tilings here are repeat patterns. However, the original are obviously finite and in some cases, the repetition used is unclear.

The more complex Islamic star patterns could be represented in slightly different ways and yet still be the same 'tiling'. One needs to try to recover the original concept which has typically been lost. Roughly speaking, the aim is to increase the symmetry, either of the tiling as a whole, or of individual tiles. It is also sometimes possible to re-use an existing tile in a pattern rather than introducing another similar tile.

In Book VIII of Plato's 'Republic', Socrates illustrates the concept of reality and what we perceive by an illustration of men who are chained inside a cave with a view exclusively of the inner wall, on which they see shadows of objects paraded behind them, obstructing the daylight. "Such men would not perceive truth to be anything other than the shadows of artificial objects." Here, we are trying to reconstruct the original idea using modern technology. Hopefully, more than a shadow…

4.3   Structural image data

The transcription process produces the graphic detail from which a drawing program can produce the images seen in this collection. In addition, structural detail is obtained, partly by computing data from the graphical representation and partly from additional information stored with the image.

One structural item is the largest angle for which all internal angles of every polygon is a multiple. This angle is computed from the image data and hence

should not be in doubt. Some patterns have polygonal angles which are effectively arbitrary — and hence the computation does not provide this measure. The 'angle' is then recorded as zero.

Another structural item is very different. This is the symmetry group. Note that we are using the Conway-Thurston notation, but retaining the conventional notation in brackets (for an explanation, see PDF). For technical reasons, the image processing software is not clever enough to compute the group. Hence the value is added by hand. This implies that the possibility of an error in assigning the symmetry group cannot be ruled out. Some simple computer checks can be made, but errors could still occur.

Some items are computed, but cannot always be determined exactly. For instance, with a repeating pattern, the computation of whether it can be coloured with just two colours is straightforward (there are an even number of lines at each vertex). Unfortunately, for centered patterns in which the fitting rules are not one-to-one between edges, the computation is not straightforward and is not undertaken.

The conclusion from the above is that the structural image data is not perfect, but with care, is quite sufficient to locate almost all patterns in the collection.

4.4   Identification of a tiling

The results for any tiling also gives a reference to the original data in the form `data`n`/name`. It is probably best to use this format to refer to a pattern for future use [1]. A URL for a pattern can be constructed by prefixing `data`n`/name` with `http://tilingsearch.org/HTML/` then appended with `.html`. Such a URL will only be changed if a significant revision is needed to the entry.

Unlike conventional books, this system is subject to change. Errors will be corrected, and as further information becomes available, the colouring of many tilings could be changed. Hence referring to a pattern as, say, Bourgoin Plate 118 may well be best! Note that an individual tiling may occur in several publications, but this system typically has a title for the tiling which refers to just one publication or location.

5   Attributes for each pattern

The database retains properties of each pattern which is used for searching. The main attributes are listed here in order of importance.

5.1   Geometric attributes

For these attributes, there are two distinct cases: when a full mathematical analysis has been undertaken and when such an analysis has not be done.

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[1]Sometimes a pattern is deleted, which only happens if a duplicate is found; then a geometrically similar could be located.

Star polygon. The number of points to the regular star is recorded and used for searching. The vertex angle is also used in some contexts. The 8-pointed star with a vertex angle of 90° is called a *khatem*. When the term "regular" is used in quotes, this implies that the star polygon is visually regular, but no analysis has been undertaken to verify this.

Regular polygon. The number of sides is recorded. When the term "regular" is used in quotes, this implies that the polygon is visually regular, but no analysis has been undertaken to verify this.

Symmetry group of tiling pattern. This is recorded, but if you are not familiar with the 17 planar groups, then it is probably best to use the tree search since that is based upon simple properties such as rotations and reflections.

Can the pattern be coloured with just two colours? The system records the property as true, false, or don't know. In some cases, the software cannot determine the value, but this is not relevant for searching, since in those cases, the attribute is not used.

The tiling angles. For the majority of tilings, all the internal angles of the polygon are multiples of a small angle. Only defined if a full mathematical analysis has been undertaken.

Edge-to-edge property This property is false if a straight line edge of a polygon is joined to more than one other polygon. This property is true or false.

Contains diamonds. A diamond is a four-sided polygon whose sides are of the same length (but not a square).

Contains a kite. A kite is a convex four-sided polygon whose edge lengths in cyclic order are AABB, where A and B are not equal.

Contains a dart. A dart is a concave four-sided polygon whose edge lengths in cyclic order are AABB, where A and B are not equal.

Some more complex attributes are explained in [6].

5.2   Other attributes

Location. When a location is known, this is specified. Details of the locations are to be found from: HTML).

Publication. These are reference by a string in square brackets as listed in see HTML).

Links to photos and documentation. There are two web sites that are reference via links: Patterns in Islamic Art and the Victoria and Albert Museum collections web site. The two sites contain additional information apart from being a source of the pattern.

Links to related patterns. There are 19 small collections of patterns that are grouped together which are to be found from: HTML). The page for each pattern links these together.

6 Searching

6.1 The tree search facility

This is probably the easiest tool to use to locate a tiling.

If you are not familiar with the symmetry groups, this search method should be suitable; however, it is slower to use than the search facility specified below. It works like a Flora — questions are asked about the tiling which leads to another question. Eventually, the number of tilings which satisfies the answers to the questions is reduced to a small enough number to display all the tilings. From this display, a final choice can be made (or you have showed that pattern being sought is not in the collection).

The questions are fixed which means that if a mistake is made, the browser *back* command allows one to retrace ones steps and make an alternative choice.

At every stage examples are given of patterns which satisfy each of the conditions associated with the new branch in the search tree.

Start a tree search by clicking here.

6.2 Search by simple options

It has been noticed that searches have been made for patterns by using the text search option in older search facility. This is not always effective (due to spelling issues) and is slow.

Hence a new fast search is now available using simple options. These options allow for the absence of a characteristic as well has the presence. Hence on can request a pattern with 6-pointed stars, but no equilateral triangles all in Egypt. There are 65 of these.

This form of search comes in four forms: a simple search, general search, advanced search, and all search. All these forms are the same except the number of options provided.

6.3 Search by location

A completely different form of searching is by location rather than geometric properties. This facility does not cover the outlying aras of the Muslim world, such as France and the UK. The searching can start from: here

6.4 Not found?

If you cannot find the pattern, please email me with the details - a photo is necessary. Brian Wichmann

7 Browsing

Four methods are provided to browse the collection:

1. Twenty randomly selected patterns can be displayed. This provides a rapid means of seeing some of the collection. The twenty examples will all be distinct, but if another 20 are requested the two batches may contain a common tiling. (You can place the URL obtained from using this facility as a bookmark to produce such a random collection.)

2. You can inspect the small collections of similar tilings as noted below.

3. You can inspect the statistical summary here. For each entry, a link is provided to a single instance of the property presented in the tables.

8 The special collections

Some types of pattern require different treatment in some circumstances. These are placed in special collections. Currently, there are four special collections as follows:

1. Tilings of rectangles by polyominoes. For details of these see here.

2. Examples of perfect colouring. For details of these see here.

3. Examples of tiling of Unique Factorization Domains. For details of these see here.

4. Examples of tiling of a square by similar triangles. For details of these see here.

5. Examples of Tilings with approximate geometry. For details of these see here.

6. Examples of Spiral Tilings. For details of these see here.

7. Examples of Lattice Tilings. For these, the star polygon are made of kites, see here.

9 Similar tilings

Having found a tiling, if there are similar ones in a small collection, these can be found by links in the page associated with a tiling. These small collections can be found directly here.

10   Islamic tilings by area of origin

Many Islamic tilings appear in several locations. These locations have been divided into regions so that it is possible to display all the tilings from a region. For the details, see here.

11   Tilings that are not included

If an individual tile is not a topological disc (for instance, it has an internal hole), then the software used here cannot handle it and therefore the pattern is excluded from consideration in giving a high quality graphic. A few patterns appear with approximate, as noted above.

Some tilings include curves. The software does not handle curves, but a sequence of lines can be used instead. Patterns which are mainly curves are not considered for a high quality graphic, but a few examples appear with approximate geometry.

A more technical restriction applies to the individual tiling which must have local connection rules [4]. This implies that Penrose tilings cannot be handled, although quite a large patch of such a tiling is possible [1]

12   Other issues

For some patterns, documentation has been produced which shows how the patterns was constructed. The file containing this documentation are not being placed on the web, but I am happy to provide them by email if a request is made. I can also provide SVG for any pattern. This would include the dynamic build, but that can be easily edited out.

From version 27, specific patterns made contain a link to a page containing a request for a photo, see Request page. This request appears for those patterns for which no photographic record is available. Such a record is vital for Islamic patterns, since it is the only way we can be sure that our pattern is correct.

The other major restriction is that the tiles must be fitted together by means of local connection rules, that is, every edge has a matching edge. This does allow tilings which have a centre and hence are not repeat patterns. It does not allow substitution patterns like to one due to Penrose [2]. A small patch of a Penrose tile is possible [1], but examples are not currently included in the collection.

13   Acknowledgments

This site is actually hosted by Perspective Technologies Ltd, the Directors of whom are the two youngest sons of the author of this system.

The list of sources shows the large number of people who have effectively contributed to this work via their publications. People who have contributed includes (alphabetically):

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References

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[2] R Penrose: Pentaplexity. *Mathematical Intelligencer*. 1979. pp32-37.

[3] B. A. Wichmann, The World of Patterns, CD and booklet. World Scientific. 2001. ISBN 981-02-4619-6 (This publication has over 4,000 patterns but it a different format from the on-line system.)

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