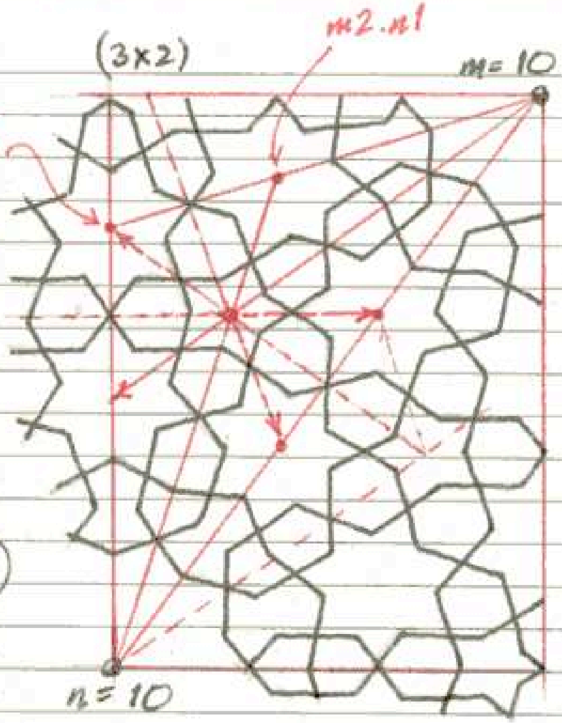
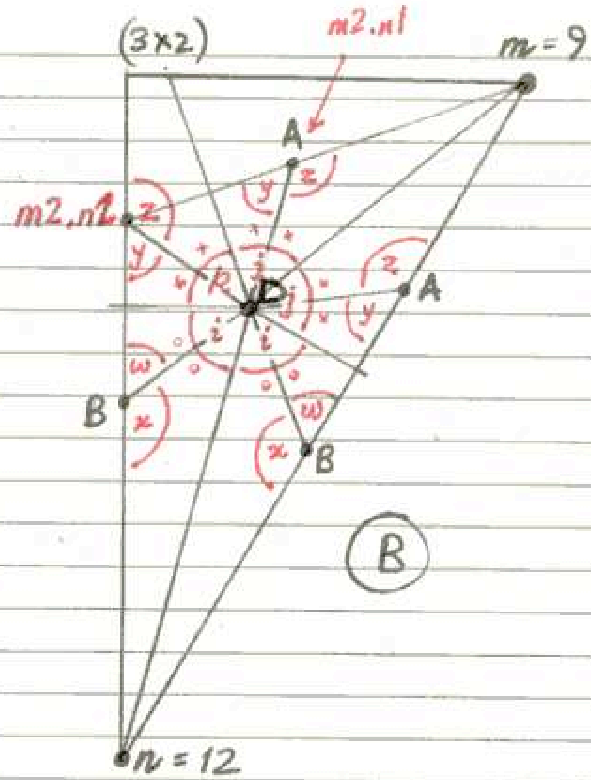
Analysis of a new (3×2) Pattern Type

Fri 27 April 1984

This falls into group "A" of pages 83, 84.





The pattern shown in fig. A above has been adapted to a (3×2) rhomb size from a similar pattern on the Tower of Masud III, Ghazni (Afghanistan) – see Hill & Grabar, 1964, fig. 146, upper right. In pattern A each 10-star is surrounded by ten nearly-regular 7-stars, with an additional pair of interstitial 7-stars at each rhomb centre. The centres of the 7-stars can be obtained by bisecting angles i, j and k in fig. B above, to give points A, B. The interstitial 7-star has its centre at the intersection m2.n2, and that between the latter and the m-centre has its centre at the intersection m2.n1. The pattern is adaptable to other m, n values in the (3×2) rhomb series, but with stars much less than 10 it will be found that 7-stars do not produce a good fit (nor a lot greater than 14, where fourteen 7-stars form an exact fit), so we must determine general expressions for the angular sizes of angles w, x, y and z (fig. B, above). These are given to the left of fig. C, opposite (p. 102).

In fig. C pattern A is adapted to (3×2) 9, 12. On the basis of the expressions for w and y it is found that 7-stars

Fri 27 April 1984

$R = \frac{m-2}{2m}, \frac{n+2}{3n}$

$j = R$

$i = \frac{m+6}{4m}, \frac{n-2}{2n}$

$w = \frac{i}{2} + \frac{1}{n}$

$= \frac{3m-6}{8m}, \frac{n+2}{4n}$

$x = 1 - w$

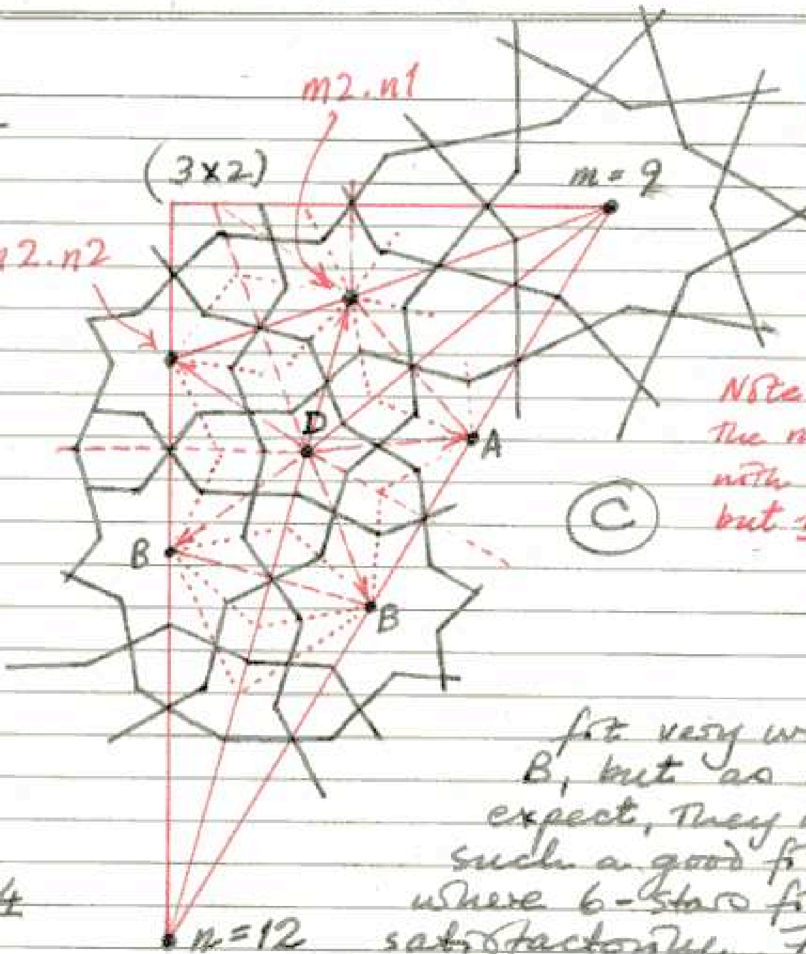
$= \frac{5m+6}{8m}, \frac{3n-2}{4n}$

$y = \frac{i}{2} + \frac{1}{m}$

$= \frac{m+14}{8m}, \frac{5n-14}{12n}$

$z = 1 - y$

$= \frac{7m-14}{8m}, \frac{7n+14}{12n}$



(original – first drawn 15 Mar 19

Note that point B
the n-axis coincide
with m1.n2 in (3x2)
but not when m ≠ n

fit very well at points B, but as one would expect, they do not form such a good fit at points where 6-stars fit much more satisfactorily. For this reason 6-stars are drawn in fig. C at points A and interstitially at intersection m2.n2.

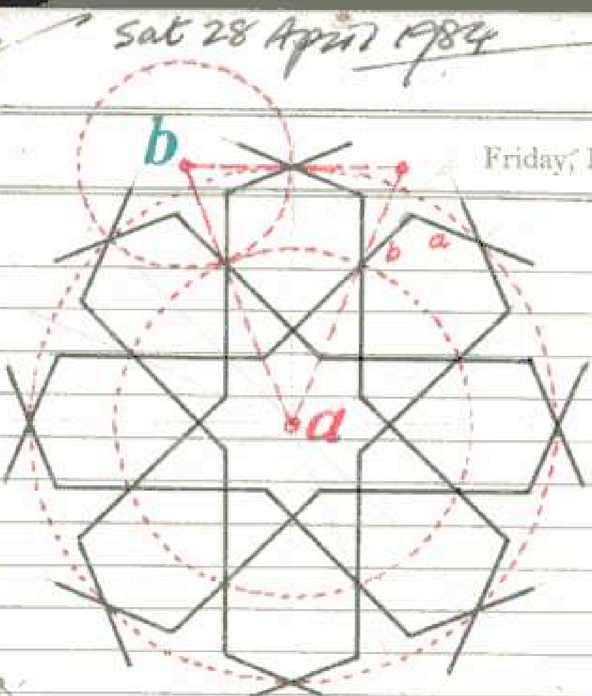
To ascertain whether or not a ring of S-stars will fit round an n-fold centre, the method outlined on pp. 19, 20 above may be used. The curves on p. 20 relate only to exact, integral solutions, but non-exact possibilities may easily be deduced from those results.

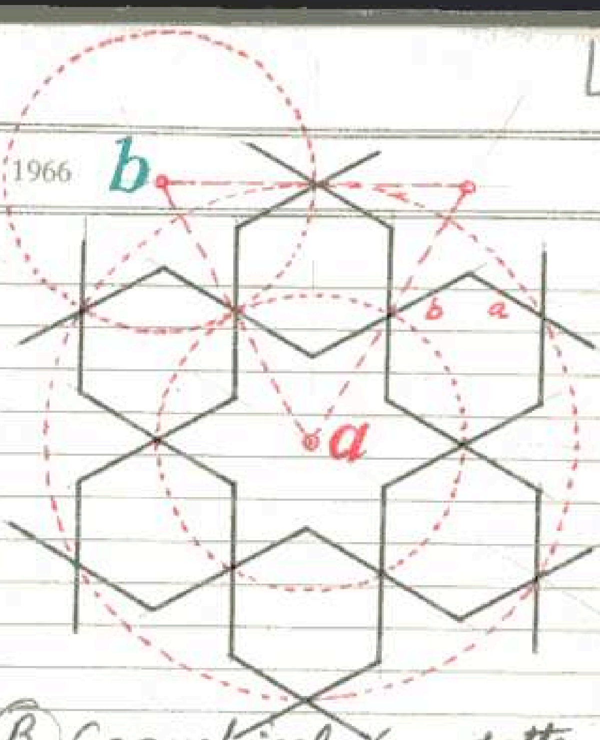
The point of the above analysis is that a more complex pattern which may well suit the central pair (i.e. m = n) of values of a (p×q) series, may not always adapt to other pairs of values without a certain amount of alteration, for example changing 7-stars to 6-stars if a better fit is thereby obtained.

Sat 28 April 1984

Extensive and detailed study of Islamic geometric ornament soon convinces us of the interrelatedness of many aspects of these patterns, both on a purely geometrical level, also with respect to possible lines of logical development and influence at the practical level. But it does not make any easier the task of writing up a coherent account of even the star patterns, since there are so many alternative logical pathways one might choose among the enormous amount of original data, in order to demonstrate ways in which specific aspects of pattern construction, styles of realization, and so on, might have evolved from one another. In order to try to deduce what might have occurred historically, on a practical level we can only make inferences, in the absence of direct historical evidence, based on geometrical similarities and relationships. With respect to the ubiquitous <u>geometrical rosette</u> (fig. A p.104), as I have suggested elsewhere it seems likely that the generalization emerged from the 6-fold case shown in fig. B (p.104).* The earliest known example of the latter as a distinct motif occurs on the tympanum of the main façade of the Arab-Ata mausoleum at Tim, in Uzbekistan (dated 977-8 A.D., Pugachenkova 1963). An outline drawing using the same proportions is shown in fig. C, opposite (other, later examples sometimes use different proportions, and therefore, one assumes, different methods of construction). This "prototypical" 6-fold rosette consists of 6 regular hexagons surrounding a 6-pointed central star (in higher rosettes the hexagons are symmetrical, but no longer regular, fig. A). Such a configuration occurs repeatedly in the pattern of 6-stars on the vertices of the regular tessellation of equilateral triangles (fig. A, p.106), but in this case each hexagon is "shared" by three adjacent stars. In the Arab-Ata pattern each configuration of 6-star and six regular hexagons is distinct from nearby configurations and so constitutes a distinct motif, in the form of a flower like, six-petalled rosette. In view of the derivation of this motif we are justified in regarding it as the prototype for the later, prevalent geometrical rosettes which occur in geometrical ornament throughout Islam, forming a characteristic Islamic decorative innovation.

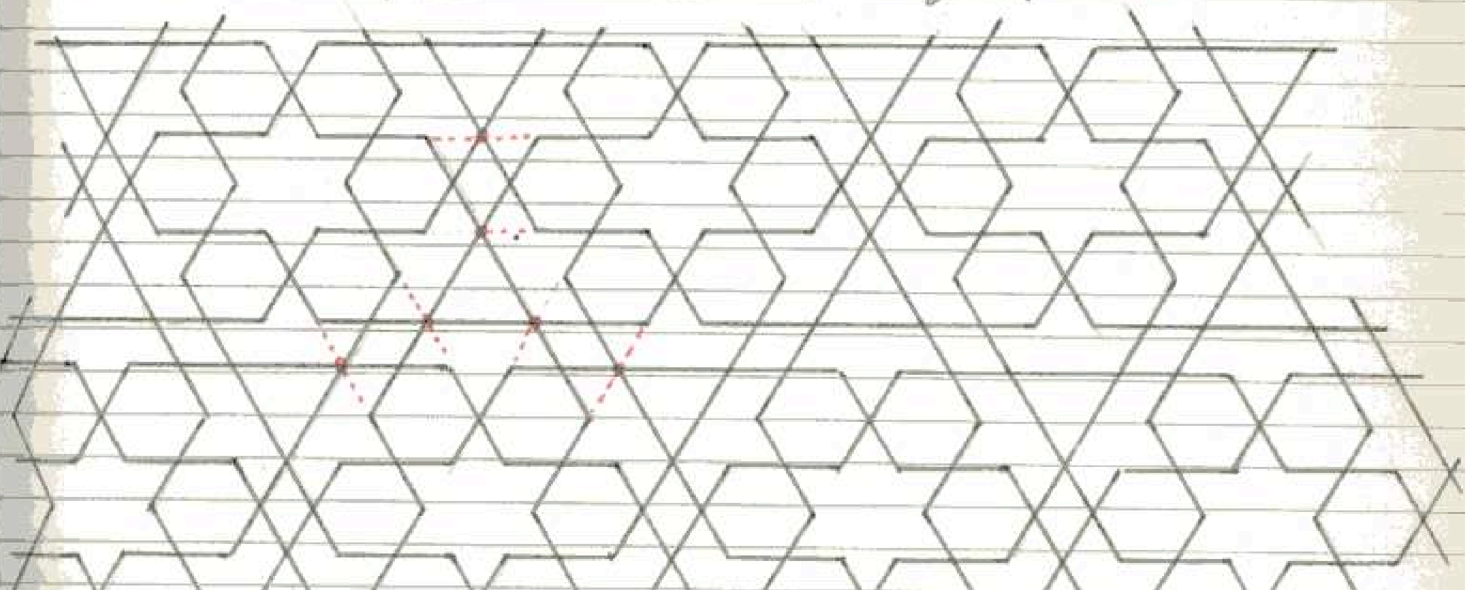
* see also p. 143 infra.





(A) Geometrical 8-rosette (B) Geometrical 6-rosette

Both with parallel-sided rays and collinear terminal segments. These properties follow automatically from the method of construction (red), furthermore, segments a and b are always equal.*



(C) Pattern incorporating prototypical 6-ray "rosette" from Arab-Ata mausoleum at Tim, Uzbekistan (977-8 A.D.).

* If we adopt it as a general rule that a = b always, then the slopes of these two lines are obviously interdependent, and indeed this also determines the slopes of similar lines in an adjacent rosette. However, differently numbered rosettes cannot both share properties such as collinear terminal segments

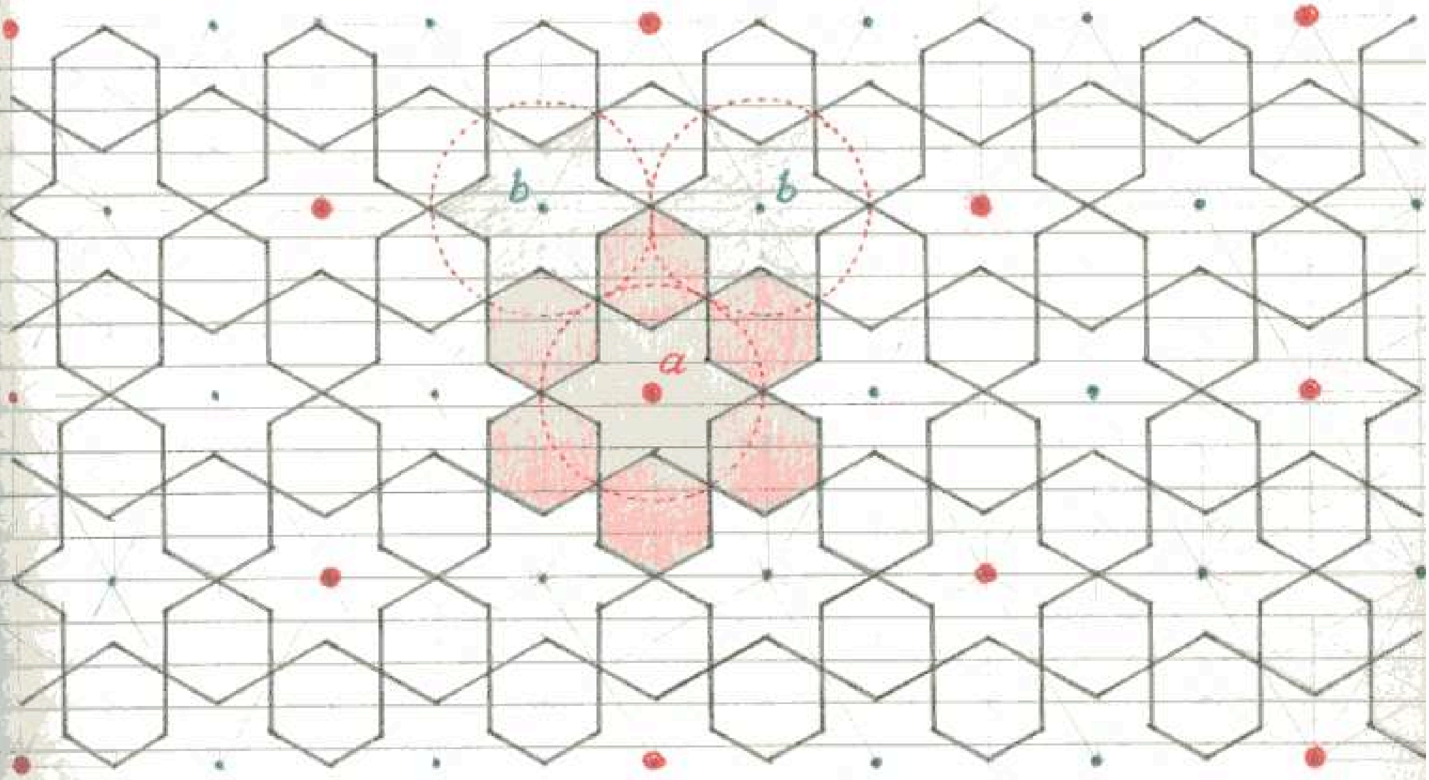
After Sat 28 April 1984

The actual historical origin of the general n-rayed geometrical rosette is not known, that is to say the time and the manner in which the generalization was actually made from the 6-fold to the n-fold condition is unknown, neither do we know the geographical region in which this innovation was instigated. If the process began with such a pattern of prototypical 6-rosettes as that at Tinn (fig. C p.104) then the generalization must have involved not only the geometrical construction illustrated in the comparison of figs. A & B (p. 104), but also the ability to visualize the new n-rayed rosettes making contact once again, without losing their individual identities (as would actually happen if the 6-rosettes were brought into contact at their outer points — producing once again the pattern of interminglied stars and regular hexagons from which we have formally derived the prototypical 6-rosette itself). Because the impetus to generalize the 6-fold case must have involved also a desire to increase the variety of the available geometric patterns. It is unlikely that the general n-rayed rosette was discovered merely as a geometrical exercise, and it seems unlikely that the primary aim was to use isolated rosettes as decorative emblems (although this practice is encountered, it is by no means as common as the incorporation of rosettes and other types of motifs in repeating patterns).

The initiating factor in the chain of events leading to the discovery of the n-rayed geometrical rosette must have been the ability to "see" 6-fold rosettes in the common and well known pattern of 6-stars (star of David) and hexagons, as emphasized in fig. A (p. 106), followed by the realization that this could form an attractive motif in its own right, and the necessity to separate these motifs in a pattern, in order to make them immediately "visible" (fig. C, p. 104). Following this initial insight the discovery of the pattern at Tinn presents no difficulties, but the next stage towards the construction of geometrical rosettes with any numbers

Sat 28 April 1984

Monday, MAY 9, 1966



(A) 6-stars on vertices of {3,6}. Those centres marked red form the vertices of a larger {3,6} and prototypical 6-rosettes such as that at a (red hexagons, yellow star) can be picked out, centred on these red points such that they contact neighbouring rosettes at their outer poi[nts]. The remaining 6-stars, centred on the green points then function a[s] peripheral stars to the primary rosettes.

of rays is not easy to ascertain. There was probably an early attempt to adapt the newly discovered motif to a square array, but strict analogy with the original pattern of 6-stars on this basis would lead only to a version of th[e] old star-and-cross pattern (fig. A, p.108), in which the "crosse[s]" are the 4-rayed "rosettes", while the "stars" (Khatem) are the peripheral elements. This pattern was already in existence, and there is no possibility of knowing whether this was ever interpreted as equivalent to an arrangement of 4-rayed "rosettes". Possibly geometrical rosettes were constructed with 5, 7, 8, 9, etc rays, but the smallest of these new rosettes which would have formed a simply

After Sun 29 April 1984

Tuesday, MAY 10, 1966

repeating pattern is the 8-rayed version (fig. A p. 104; fig. B p. 108). Patterns are of course possible with 5-, 7- and 9-rayed versions, but they leave substantial gaps or residual spaces that need filling with some additional interstitial elements, and may have been thought unsatisfactory for this reason. The pattern of 8 rosettes leaves very little interstitial space and this becomes automatically filled by simple regular octagons.

It is of course conceivable that the impetus towards the discovery of the general n-rayed rosette could have been initiated through contemplation of the star-and-cross pattern (fig. A p. 108) — or perhaps from a combination of both this pattern and the six-star pattern (p. 106), — but it seems rather less likely, since the conceptual jump appears greater from the 4-rayed "cross" to the general rosette than from the 6-rayed "rosette", which indeed has much more the appearance of a prototypical rosette.

8-rayed rosettes form a very satisfactory pattern (fig. B, opposite), and the next number to produce a suitable pattern is also an even number, and concerns the 10-rayed rosette (fig. A, p. 110). A little trial and error would have been necessary to discover this pattern, since the triangular and square grids used for most geometrical patterns are unsuitable (there is of course no question of placing 10-fold motifs on n-fold rotocentres where n is a multiple of 5, but the earliest designers of patterns, or indeed the contemporary mathematicians themselves would not in any case have thought in terms of rotocentres). The steps leading to the discovery of this exact pattern — granted the initial knowledge of a 10-rayed rosette — cannot be known, and would not have been preserved, but this pattern is in fact the only possible arrangement* in which the rosettes meet at their outer points and nowhere overlap one another. We should note the geometrical elegance of this pattern with regard to many of its details. Many line segments of various slopes form collinear sets extending various distances through the pattern, and in some cases the

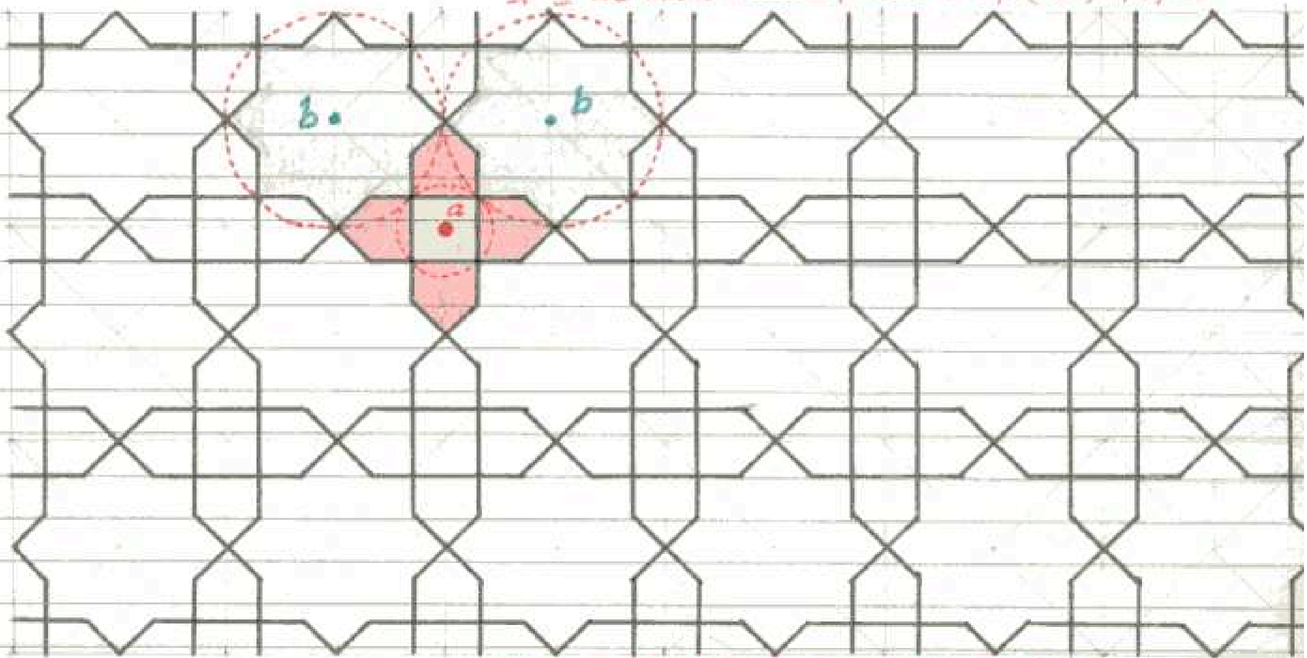
* see note on p. 109

Sun 29 April 1984

Wednesday, MAY 11, 1966

There is also a solution with peripheral 12-stars and 3-fold 'rosettes': see p. 190. A.

If a are main "rosettes", this is Sp1(1×1)4,4/II

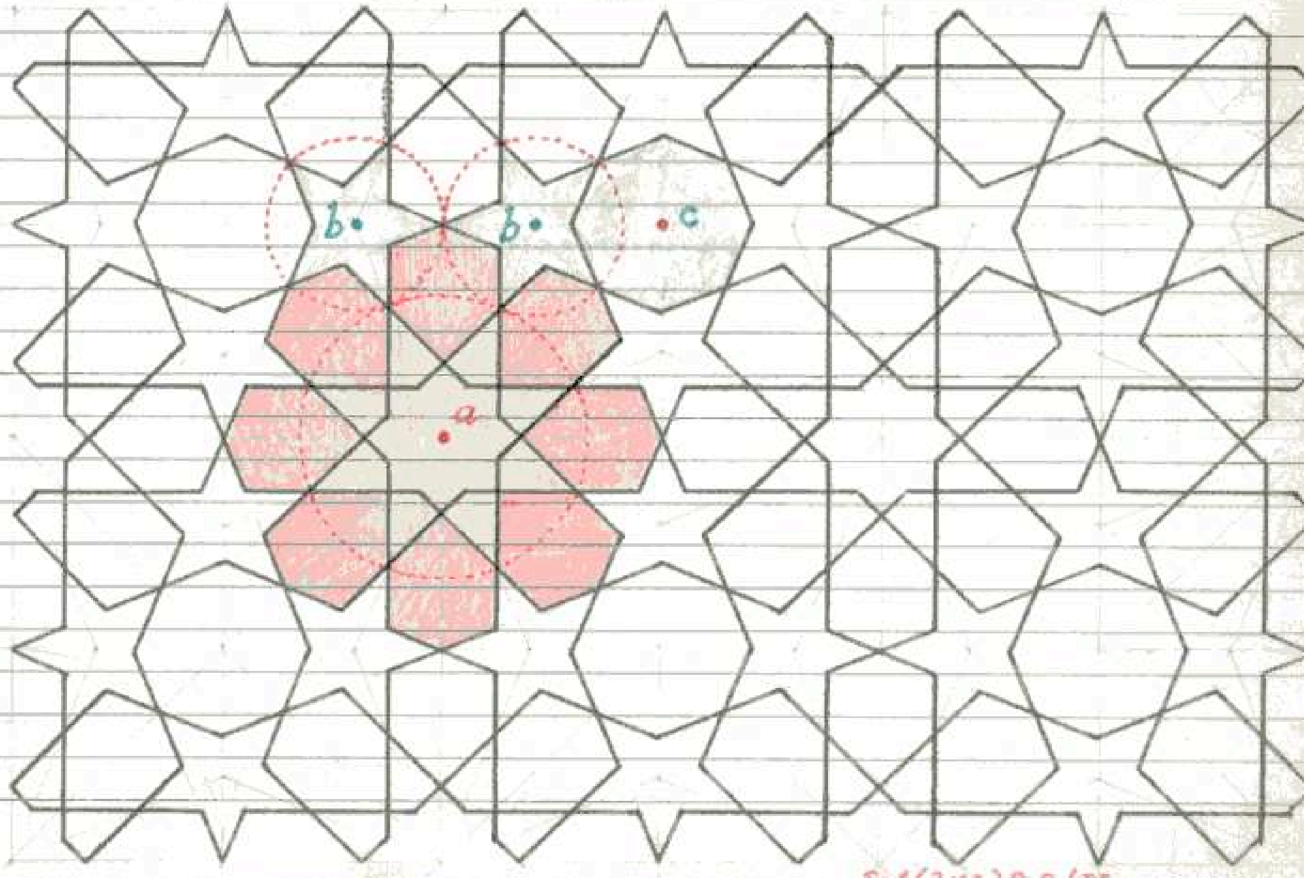


A

If b are main stars, this is Sp1(2×2)8,8/I

b = peripheral stars ; a = 4- or 8-rayed geometrical rosett

c = interstitial elements



B

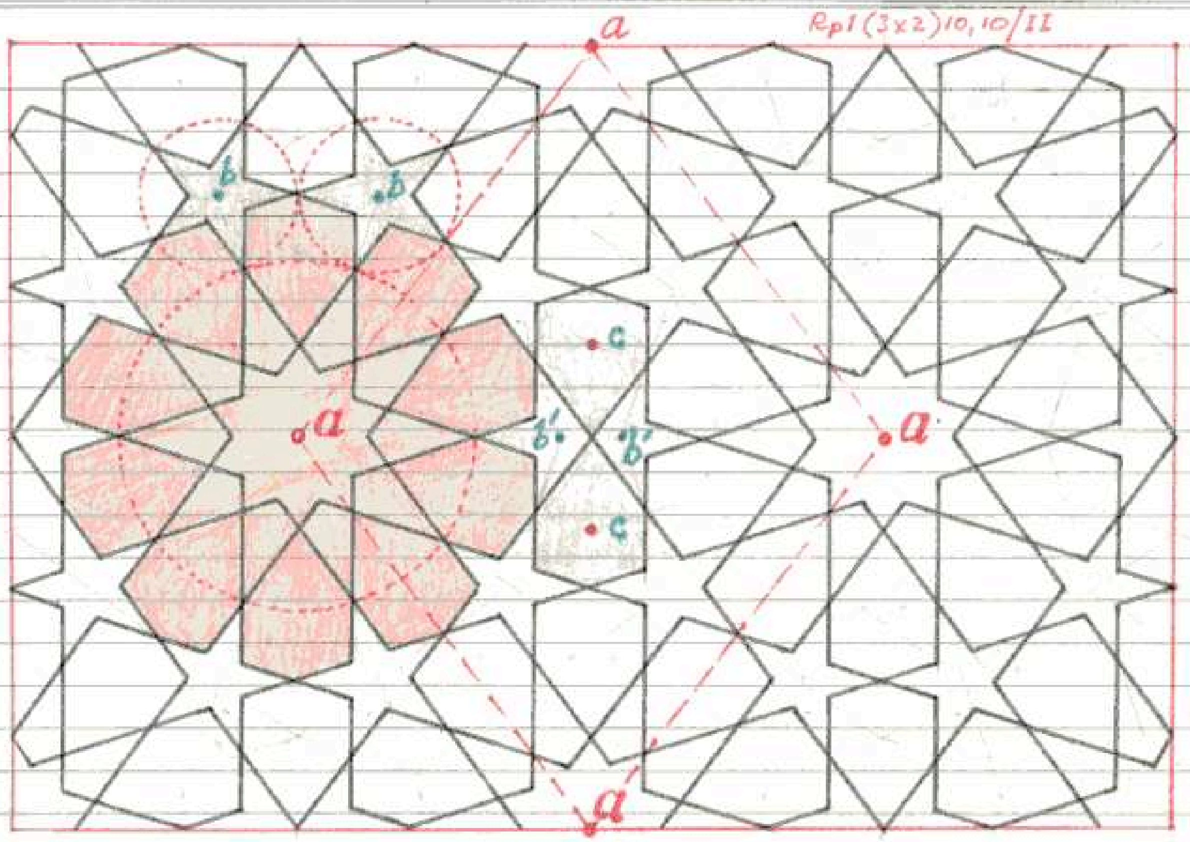
Sp1(2×2)8,8/II

Sun 29 April 1984

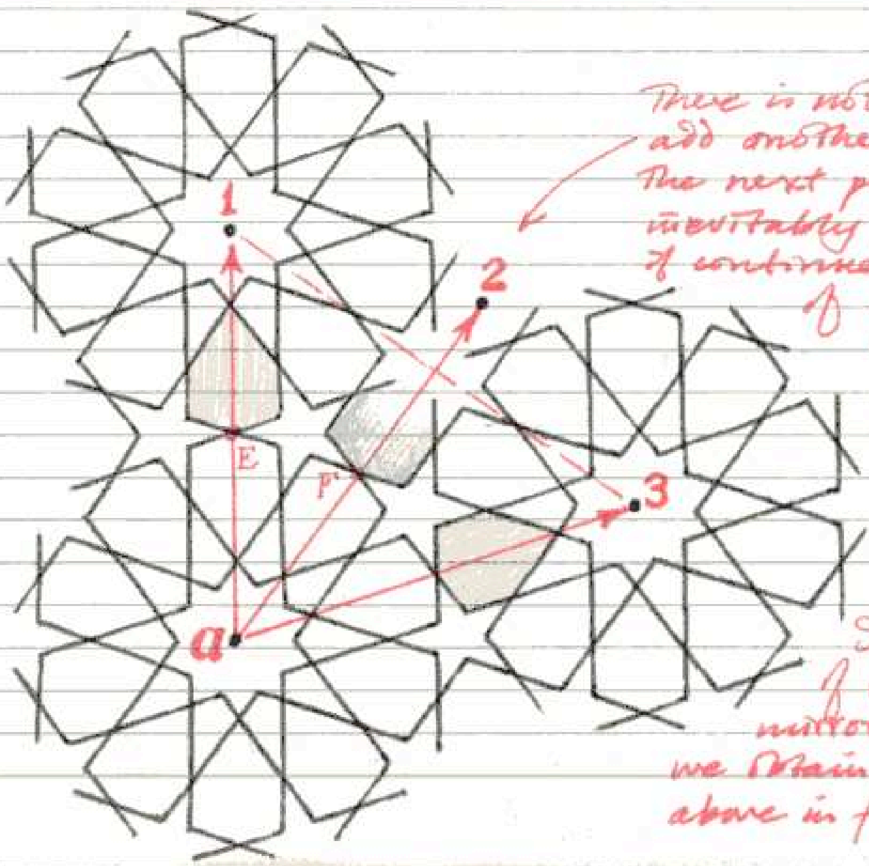
collinearity extend to infinity. The peripheral stars (b) are regular pentagrams, and the interstitial pattern (c) consists of a pair of cells (blue) which are congruent to the outer cells (red) of the main rosette (a). The points of the interstitial cells meet exactly at the centre of the rhombus outlined by the centres of the four rosettes which surround them.

* Note: from p. 107:—
Starting with a 10-rosette, a in fig. 110B, opposite, we add a second rosette at position 1 so that the rosettes meet at their outer points and form a collinear link between their centres. The next radius clockwise from rosette a is number 2, but a ~~third~~ rosette cannot be added here, since there is not enough room for a complete rosette without overlapping the 2nd, at position 1. The next possible position for a third rosette is at position 3. This leaves a small gap between rosettes 1 and 3 across radius 2, but note the collinearity of certain terminal segments across this gap which enables us to complete the pattern by drawing a pair of interstitial cells as at fig. 110A which are congruent to the outer cells of the rosettes. Now, the process of adding more rosettes can be continued round the central rosette a, but only at odd-numbered radii, until we come full circle to position 1, and it will be discovered that this produces a ring of five 10-rosettes separated by five pairs of interstitial cells. Five of the outer points of rosette a contact the outer cell of one of the five surrounding rosettes, while the other five outer points contact five interstitial cells. This method of completing the pattern is found in the N. Dome Chamber of the masjid-i Jami, Isfahan. However, it does not by itself lead directly to an infinite plane pattern. If instead of creating a ring of 5 rosettes contacting a central 10-rosette, we mirror the arrangement in fig. 110B across the line joining centres 1 & 3, then we obtain the rhombus outlined in fig. 110A, which

Friday, MAY 13, 1966



A



There is not room enough to add another rosette here, so the next possible position is inevitably position 3: this if continued leads to a ring of 5 rosettes separated by 5 interstitial pairs, surrounding a central rosette of the same size.

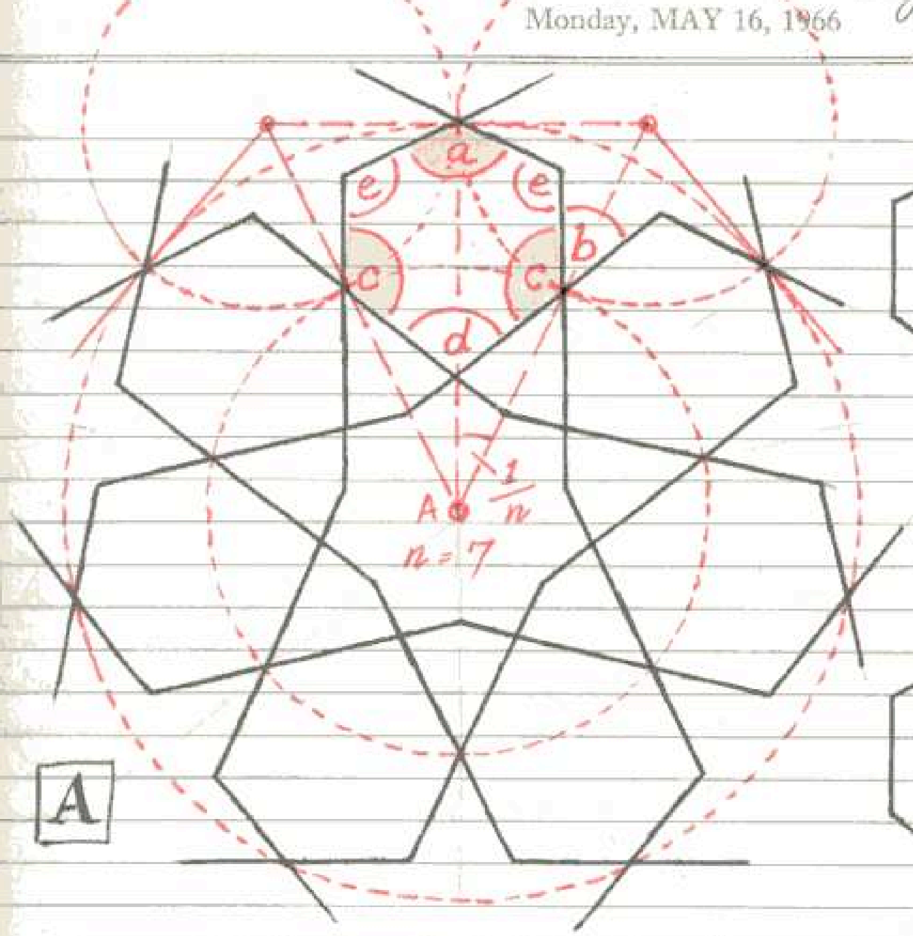
B

If the configuration of rosettes a, 1 & 3 is mirrored across line 1-3 we obtain the rhomb shown above in fig. A.

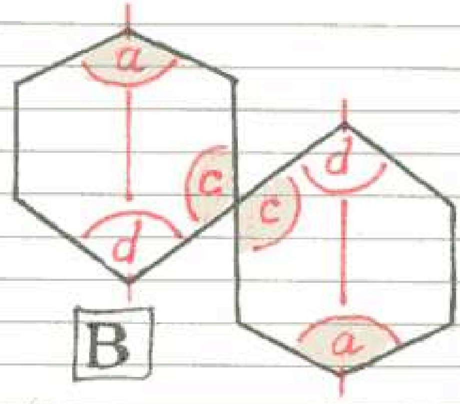
can repeat indefinitely to fill the plane. Note that both processes inevitably lead to the formation of an interstitial pair of cells which are congruent to the outer cells of the rosettes, a fact which must have deeply impressed the original discoverer of this arrangement. This leads us to wonder whether experimenting with interstitial pair configurations might lead to new discoveries with rosettes of other sizes. The original artists would have satisfied their curiosity merely by trial and error juxtapositions using the outer cells of various sizes of rosettes, but the modern student can do better than this by studying the geometry of the rosettes with parallel sides and collinear terminal segments, to ascertain general properties common to all sizes on an exact mathematical basis.

In fig. 112 A the constructed geometrical rosette (here, a 7-rosette) has parallel sides and collinear terminal segments (cf. p.9 et seq.). Because of the parallel sides, angle (b) is obviously equal to $2/n$, and because of the collinear terminal segments angle (a) is easily seen to be $(n-2)/n$, and this is also the value of angle (c). From the latter fact it is possible to link up a pair of outer cells in three ways: (1) as in the original rosette c+c (2) as in fig. B, c+c, but with the cells orientated in opposite directions, and (3) as in fig C, that is c+a. From the latter it is evident that configuration D is always possible with the outer cells of a geometrical rosette with parallel sides and collinear terminal segments, and that if a suitable periodic arrangement can be found, then this configuration represents an interstitial pair configuration. Note that only when $n=10$ do the inner points of the interstitial cells meet at point x (fig. D). When $n<10$ these inner points fall short of point x by a distance which increases as n become smaller. When $n>10$ the inner points of the interstitial cells overlap to a greater degree as n increases.

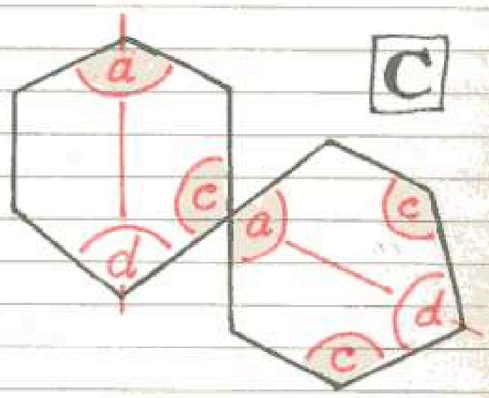
If one pair of interstitial cells is attached to a rosette, centred on point x (fig. 112D) then ~~the~~ another



A



B

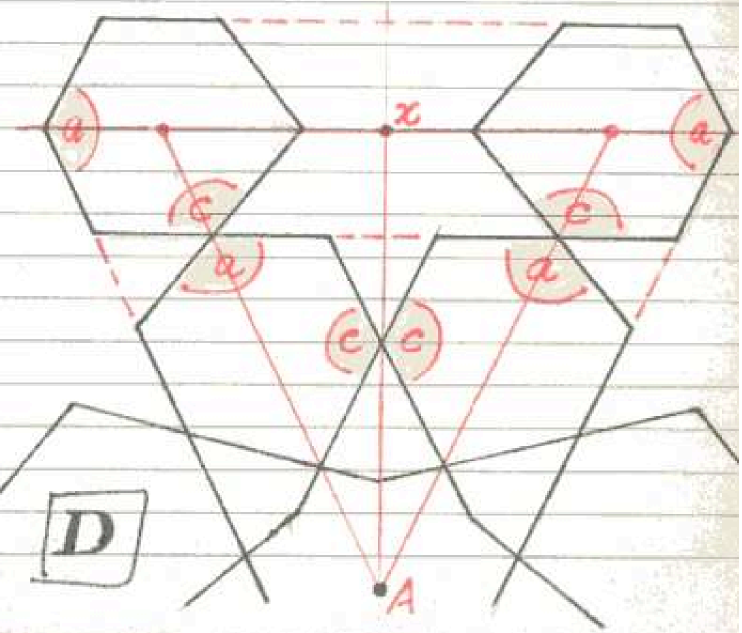


C

$$a = \frac{n-2}{n} = c \qquad b = 2/n$$

$$d = 2b = 4/n$$

$$e = 1 - \tfrac{1}{2}a = \frac{n+2}{2n}$$



D

Wed 2 May 1984

pair can be attached such the angle between the two radii Ax is no less than $4/n$, where n is the number of rays in the rosette (see fig. 114A). If the process is continued, that is, the angle $\underline{a}$ remains constant (fig. 114A), then a ring of rosettes may or may not be built up. Angle $\underline{b}$ is obviously equal to $\frac{n-2y}{2n}$ (y being the number of divisions $1/n$ in angle $\underline{a}$), and if this is reducible to the form $1/m$, then a ring of $\underline{m}$ n-fold rosettes is possible, alternating with interstitial pairs. The table below shows possible value of $\underline{m}$ for different sizes, n, of rosettes. An interstitial pair must be placed so that point $\underline{x}$ lies on a secondary radius or interradius, and if $\underline{n}$ is even the limiting position for the two interstitial pairs is when the two points $\underline{x}$ lie diametrically opposite one another across the rosette, i.e. the "ring" is infinitely large. This limit is reached when $y = n/2$ for even numbers. In the case of odd numbers the limit is reached when $y = \frac{n-1}{2}$ i.e. $b = 1/2n$, therefore $m = 2n$

Integral solns. of:

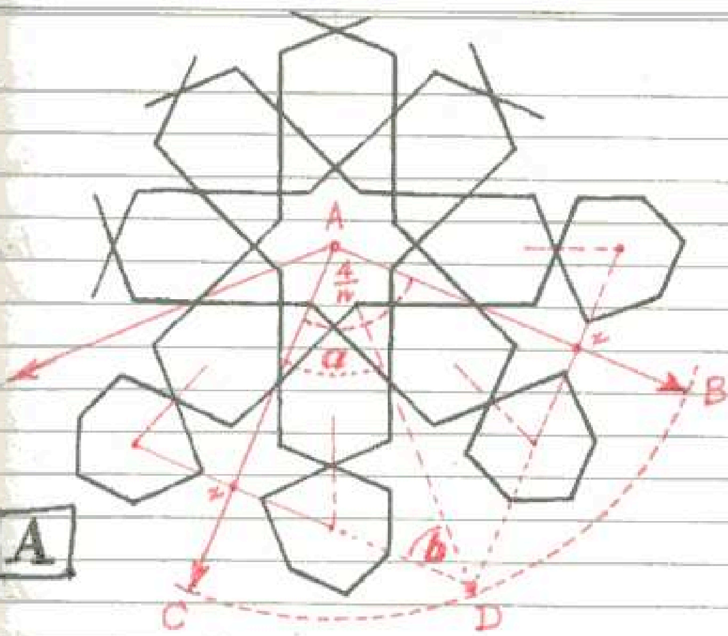
$$\boxed{m = \frac{2n}{n-2y}}$$

n \ $y =$	2	3	4	5	6	7	8	9	10
5	10								
6	6								
7	4.67	14							
8	4	8							
9	—	6	18						
10	—	5	10						
11	—	—	7.3	22					
12	3	4	6	12					
13	—	—	—	—	26				
14	—	—	—	7	14				
15	—	—	—	6	10	30			
16	—	—	4	—	8	16			
17	—	—	—	—	—	—	34		
18	—	3	—	—	6	9	18		
19	—	—	—	—	—	—	—	38	
20	—	—	—	4	5	—	10	20	

values of $\underline{m}$, when $\frac{n-2y}{2n}$ is reduced to the form $1/m$.

In the case of $y=3$ this allows a central rosette of $2m$ rays.

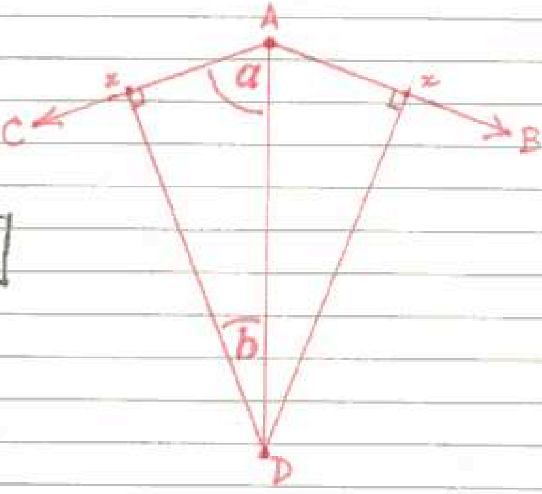
2×1 rhombs 3×2 rhombs



A

In the formation of interstitial pairs, B and C must lie along secondary or interradii of the primary rosette, but D may lie along a principal or secondary radius. Angle $\widehat{BAD} = \widehat{CAD}$ must equal to or greater than $2/n$.

y = number of divisions $1/n$ in angle $\underline{a}$.



B

$a = y/n$ where $y = 2, 3, 4 \ldots n-1$

$b = \dfrac{n-2y}{2n}$

We are interested in angles which will produce a closed ring of rosettes at centres A, adjacent pairs separated by an interstitial pair. Therefore, we must look for cases where $\frac{n-2y}{2n}$ reduces to the form $1/m$, where m is an integer. In such cases we can therefore draw a ring of $\underline{m}$ rosettes alternating with pairs of interstitial cells (the latter congruent to the outer cells of the rosettes), with the point D at the centre of the ring. The table opposite shows possible values of $\underline{m}$ for different values of $\underline{n}$ and $\underline{y}$.

After Wed 2 May 1984

However, the formation of rings of rosettes alternating with pairs of interstitial cells is only half the story. In fig. 110B we saw that a ring of five 10-rosettes alternating with interstitial pairs exactly encloses a sixth rosette equal to the others: five of its outer cells contact outer cells of the surrounding rosettes, and five contact interstitial cells. It is evident that points E, F are equidistant from the centre, a, of the central rosette (fig 110).

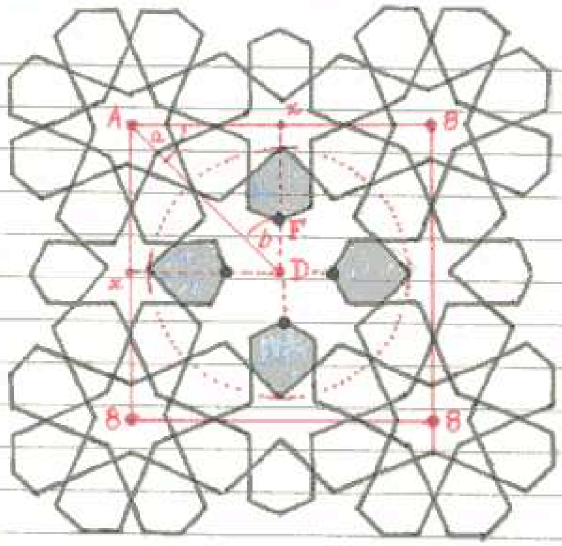
If we construct rings of 8-rosettes alternating with interstitial pairs (4 or 8 in a ring according to the table on p. 113) we obtain the two situations shown in fig. 116, opposite. In fig. A there are only four interstitial cells orientated towards the centre D of the ring and their inner points F are obviously equidistant from D. The most obvious method of filling in the space at the centre of the ring is by means of a small, regular octagon, and this is perhaps the most common method (this arrangement being in fact a widespread authentic pattern). In fig. 116B there are now eight interstitial cells and eight outer rosette cells orientated towards the centre D of the ring. Their inner points E, F are equidistant from the centre D and they are equally distributed round the circumference of a single circle with radius DE = DF. (It is also obvious that the points of these 16 cells which are the furthest from D also lie on a single circle). There is one method of filling in the centre of this ring which is staring us in the face — and it soon becomes obvious that a 16-rayed rosette would fit beautifully here. Furthermore, by combining figs A & B on one pattern we obtain a periodic pattern in which the 8-rayed rosettes are centred on the vertices of the semi-regular tessellation t{4,4}. Such a pattern is common in the Maghreb, but rare or absent elsewhere, although a type I on the same basis occurs in Iran.

We have now hit upon a general method* of pattern construction, which was almost certainly used at least to some extent by the early muslim pattern

* not perhaps important from the point of view of the number of patterns it produced, but more because of where the method leads.

Friday, MAY 20, 1966

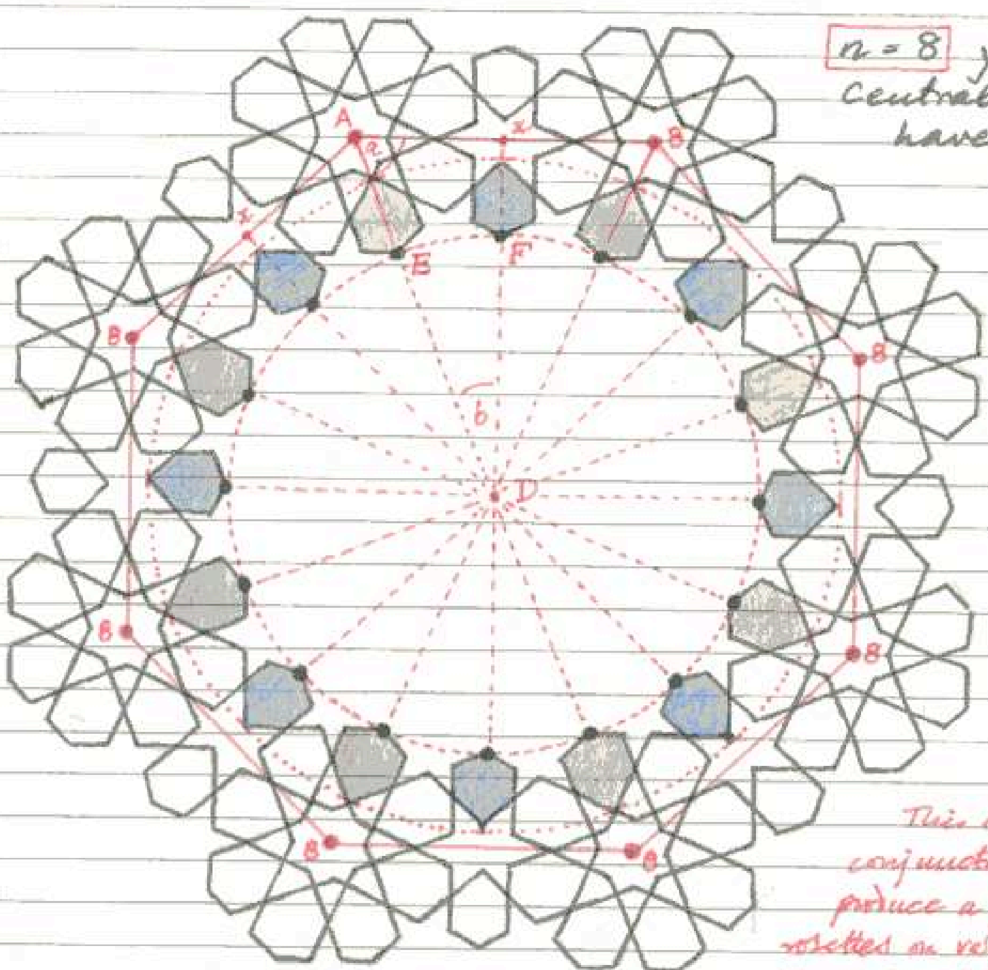
There are always m interstitial cells pointing towards the centre D, but only when y is odd do the surrounding rosettes have an outer cell pointing toward D. Only when y = 3 do we obtain the situation when ED = FD as in fig. 116B and fig. 118.



$\boxed{n = 8}$ $y = 2$ $m = 4$

A

If we regard the sides of the red square as mirrors this produces the common pattern Sp1(2x1)8,4.



$\boxed{n = 8}$ $y = 3$ $m = 8$

Central rosette will have 16 rays.

B

This can be used in conjunction with fig. A to produce a pattern with 8-rayed rosettes on vertices of t{4,4}, and 16-rosettes at face-centres of the octagons of this tessellation.

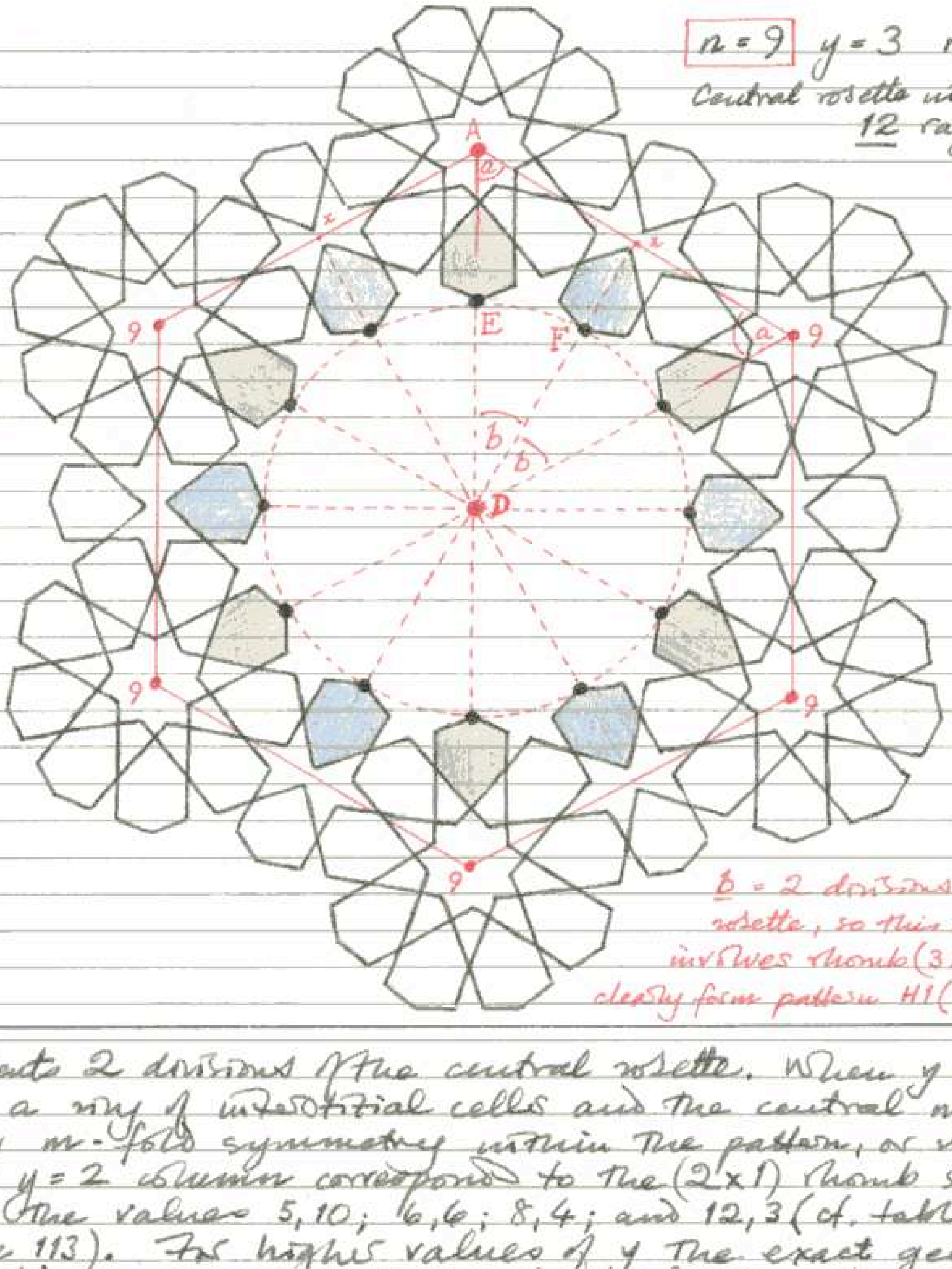
designers, after the invention of the geometrical rosette with parallel sides and collinear terminal segments (it will work for rosettes in which these conditions are absent, provided that the outer i.e. terminal, and subterminal segments of the outer cells of the rosette are equal — but this involves additional analysis and understanding of rosette geometry).
If the historical developments we have suggested actually took place then the general n-rayed geometrical rosette appeared no later than the middle of the 11th century. Mosaic shapes which could be manipulated and rearranged by trial and error were probably available at about the same time so the production of rings of rosettes alternating with interstitial pairs, and the discovery that larger rosettes could be fitting inside the rings — in certain cases — is quite feasible.

Note that the size of the central rosette is ~~such~~ related to the value of $\underline{m}$ (p. 113) such that it becomes a 2m-rayed rosette.* A ring of 6 9-rayed rosettes is illustrated on p. 118 opposite, $m = 6$ and it is obvious that a 12-rayed rosette would fit nicely inside the ring. If we recall the discussion of the general $(p \times q)\, m, n$ rhombs on p. 11 et seq. it should be obvious that these cases under discussion here, in which $y = 3$, correspond to the (3×2) rhomb series (unfortunately the designations $\underline{m}$ and $\underline{n}$ in the general $(p \times q)\, m, n$ rhomb discussion and the present examination of rings of n-rayed rosettes, are the opposite of one another — this will have to be ironed out in any definitive account), and the column under $y = 3$ in the table on p. 113 corresponds to the values 7, 28; 8, 16; 9, 12; 10, 10; and 12, 8 and 16, 6 in the $(3 \times 2)\, m, n$ series (see table on p. 14). Furthermore, all the pairs of values in Table 113 can be interpreted as $(p \times q)$ rhombs, where $y = p$, since angle $\underline{b}$ in figs 116, 118 represents q divisions of the central star centre in the ring of rosettes.
However, the correspondence here is somewhat complicated. When $y = 3$ the central rosette has 2m rays and angle $\underline{b}$

* That is, when $y = 3$; not for other values of y.

$\boxed{n = 9}$ $y = 3$ $m = 6$

Central rosette will have $\underline{12}$ rays



$\underline{b}$ = 2 divisions of the central rosette, so this arrangement involves rhomb (3x2)9,12. It a[...] clearly form pattern H1(3x2)9,12.

represents 2 divisions of the central rosette. When $y = 2$ there only a ring of interstitial cells and the central motif has only m-fold symmetry within the pattern, or ring. Thus the $y = 2$ column corresponds to the (2x1) rhomb series with the values 5,10; 6,6; 8,4; and 12,3 (cf. table 14 and table 113). For higher values of y the exact geometrical relationship of points E, F on figs 116 & 118 no longer applies, so the construction of a central rosette is not straight-forward, although the number of its rays is a multiple of $\underline{m}$.

Wed 2 May 1984

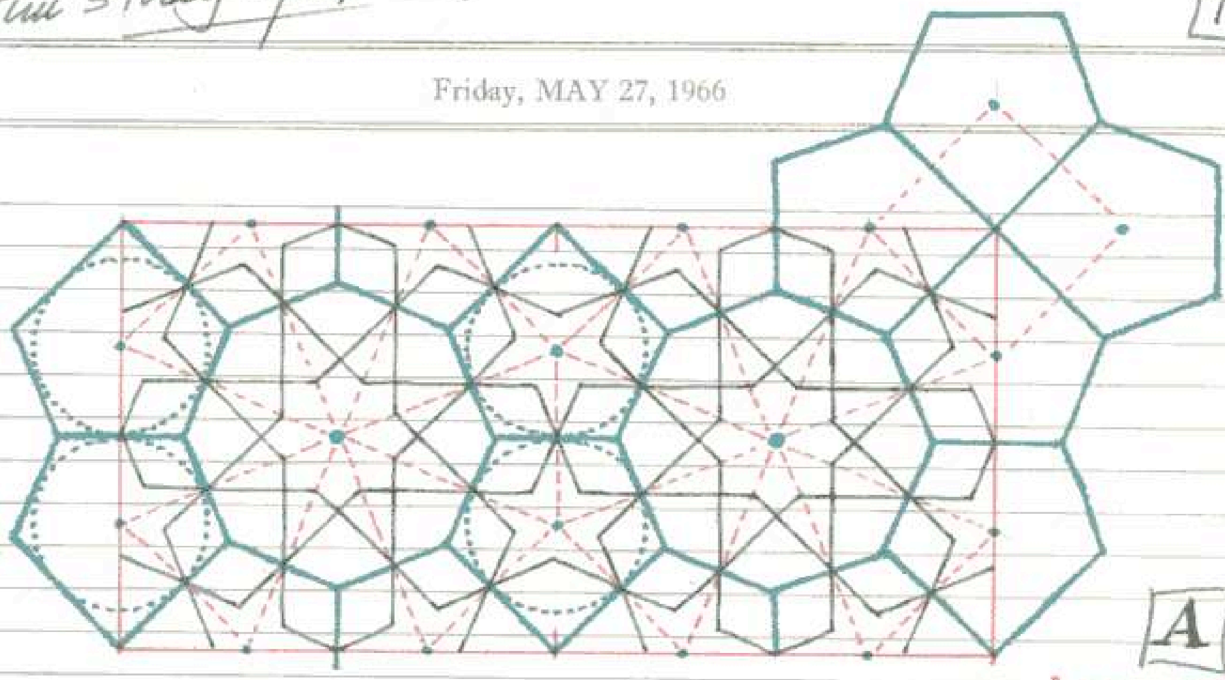
So far, by the method of constructing rings of rosettes alternating with interstitial pairs of cells we have the following pairs of values for rosettes centred on points A, D respectively: 8,16; 9,12; 10,10. These are pairs of values in the (3x2) rhomb series. The pair 7,28 would perhaps not have been discovered quite so readily by the presumed trial and error methods of the original artists, but in any case it would not itself lead to a satisfactory repeating pattern. In all these pairs of values the interstitial cells do not meet, or meet exactly at point x (see fig. 114). The remaining pairs of values in the (3x2) rhomb series – 12,8; 14,7; 18,6; 30,5 – if the rosettes on vertices A are constructed with parallel sides and collinear terminal segments, produce interstitial cells which overlap at point x, which conceivably could have momentarily discouraged any designer trying these values, although of course it is highly unlikely that these artists would have specifically paid attention only to (3x2) rhomb pairs of values, since any values tried would have been chosen on the basis of trial and error. There is thus a little difficulty in deducing the sequence of events leading to the discovery of the remaining pairs in the (3x2) rhomb series, all of which, except for (30,5) are found in authentic Islamic patterns. It seems possible however that the pair (12,8) was not discovered by the methods just outlined, but by some other means. It may be significant that many artisans appear to have had little knowledge of the correct ratio of the two lengths of radii in Sp4(3x2)12,8, many different versions being found throughout Islam. Since this pattern is on a square basis it may derive from a tradition of placing rosettes on the vertices of the square grid in which the rosette numbers are multiples of 4; 8 and 12 would therefore be among the earliest numbers selected for trial and error patterns. It is not known just when these patterns were first discovered, so we cannot estimate times at which various lines of knowledge in pattern construction and in understanding the underlying geometry may have coincided at different places. Some constructions of Sp1(3x2)12,8 give evidence of knowledge of PIC method of determining the relative radii of the two kinds of rosette, others clearly do not. Neither is it known to what extent if any knowledge of Sp1(3x2)12,8/II entailed also knowledge of the type I version of this pattern. The latter definitely requires a knowledge of PIC methods of construction or some other comparable method, otherwise the pattern cannot be drawn accurately as a straight line version. If sufficient data were available it might be possible to show that the type I version tended to be absent in areas (or at specific times also) where knowledge of the correct construction of the type II version was also absent. Type I patterns with dissimilar rosette numbers seem to be absent from the Maghreb, and here also Sp1(3x2)12,8/II appears to be usually incorrectly constructed (a version in the Alhambra seems to be correct – the 12-rosettes have non-parallel sides, which is unusual for the maghreb), but unfortunately not enough data are available for any definite conclusion to be drawn. H1(3x2)9,12 appears to have the correct ratio in the maghreb, but if the construction of this followed from the methods just outlined in constructing rings of rosettes alternating with interstitial pairs of outer cells, then the correct, PIC, ratio is automatically achieved without any knowledge of the PIC method, or any similar method of construction. If my suggestions as to the possible discovery of this pattern are correct, then all or most versions of this pattern should have the correct, PIC, ratio of radii, contrasting with Sp1(3x2)12,8 which should present many inaccurate versions, unless the PIC or similar method of construction were known (in which case of course it could have been applied to H1(3x2)9,12 also. In the absence of firm data, however, most of these suggestions must remain conjectural, although it is probably still worthwhile to give them an airing.

Thu 3 May 1984

At different stages in the general account from p. 103 we could branch off and follow up a particular line of investigation. Here we shall look up the two patterns of 8-fold rosettes illustrated in figs. 108 B, and 116 A, to show one way in which a deeper examination of certain patterns may lead to a very fruitful outcome in terms of number and variety of additional patterns. In both diagrams opposite the "linking" or circumscribing polygons of the peripheral stars and the outer stars of the 8-fold rosettes have been added in green. As a general rule the circumscribing polygon has the same number of sides as the star that points, so fig. A consists of pentagons (non-regular, but symmetrical) and regular octagons; the same octagons and pentagons appear in fig. B but we see here the addition of marginal hexagons (non-regular, but symmetrical with perpendicular axes of symmetry). The possibility of circumscribing such polygons round stars or rosettes was known to many artisans at later period, and sometimes nets of such polygons are added to star or rosette patterns, or are even used as geometrical patterns by themselves. Many of these nets have a number of interesting geometrical properties, whether or not the polygons are all regular polygons. Sometimes a drawing will suggest that a certain net has an interesting feature, which on closer investigation is revealed to be based on a mathematical inexactitude. Such is the "discovery" shown on the bottom half of fig 122 B. However, if the net is drawn correctly (that is, one that will produce a pattern in which the interstitial cells are congruent to the outer cells of the rosettes) the small "squares" are not exact - angle a is about 87° 23' and half-diagonal A is slightly longer than B. This approximation is made use of in a pattern from Iran where the diagonal distance between two 8-fold centres is spanned by two hexagonal cells, end to end.

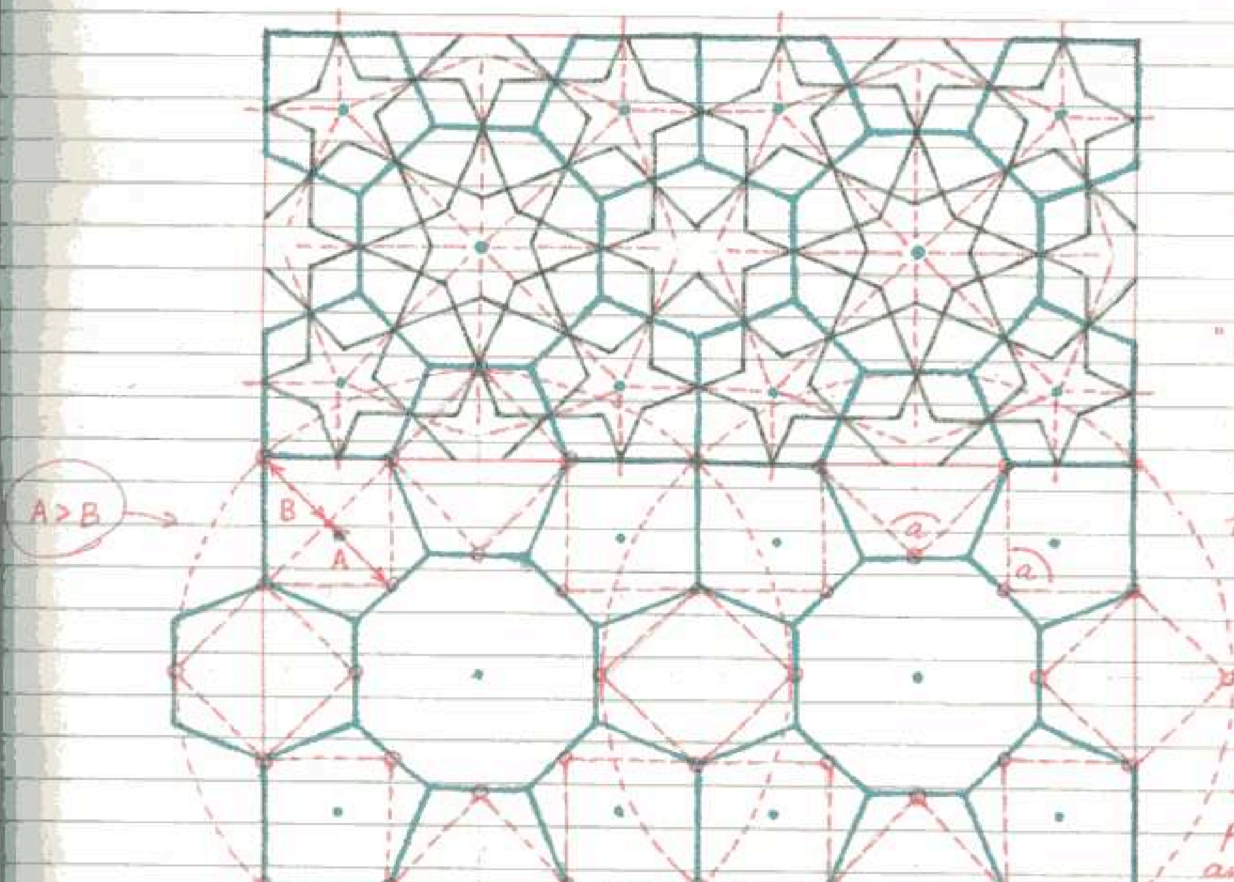
The hexagons in the net are equilateral, and the pentagons have their three shorter sides equal, but are otherwise far from regular. Different artists have tried to regularise

Friday, MAY 27, 1966



A

"5,8 not"?

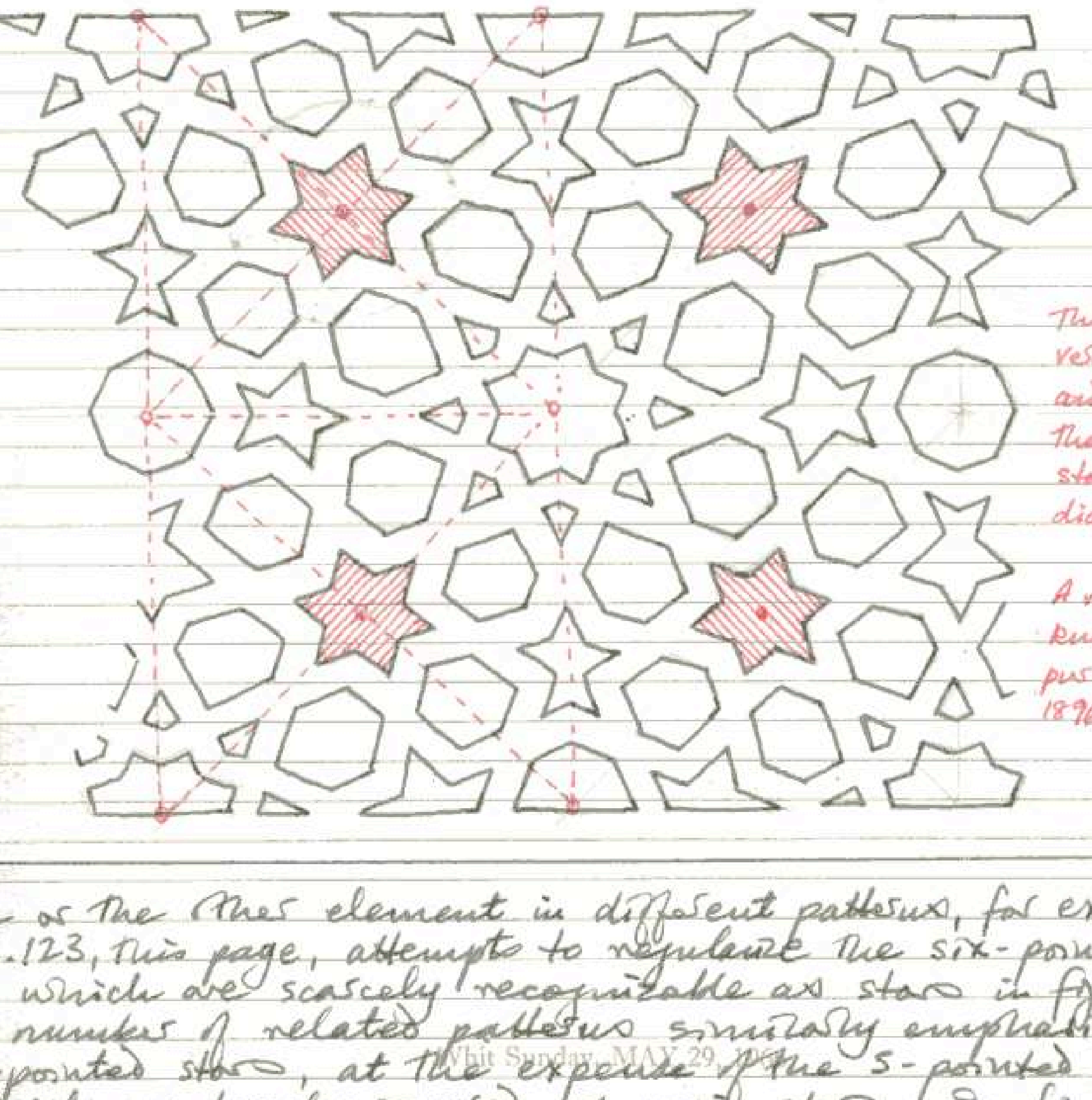


"5,6,8 not"?

B

The bottom half of this figure shows a close approximation to an overlay of small squares. In fact, in the pattern itself angle $\underline{a}$ is about 87° 23'

After Fri 4 May 1984



This Middle East version regularises and emphasizes the 6-pointed stars occupying diads.

A version also known at Fateh-pur Sikri (Smith 1896, Plate LV).

one or the other element in different patterns, for example fig. 123, this page, attempts to regularize the six-pointed stars which are scarcely recognizable as stars in fig. 122 B. A number of related patterns similarly emphasize the 6-pointed stars, at the expense of the 5-pointed stars, which are largely ignored as main stars. In fig. 124A on the other hand the 5-rayed centres are regularized. This figure is derivable from the green net of fig. 122B, the net polygons being separated from one another by a narrow gap, and stars with the same numbers of points are inserted at the centres of the polygons.

Apparently a direct derivative of the same net is the tour de force surrounding the main entrance of the Sultan Han on the Konya/Aksaray Road, Turkey – fig. 124C. Here the 5-, 6- and 8-pointed stars have been doubled to 10-, 12- and

Fri 4 May 1984

Monday, MAY 30, 1966

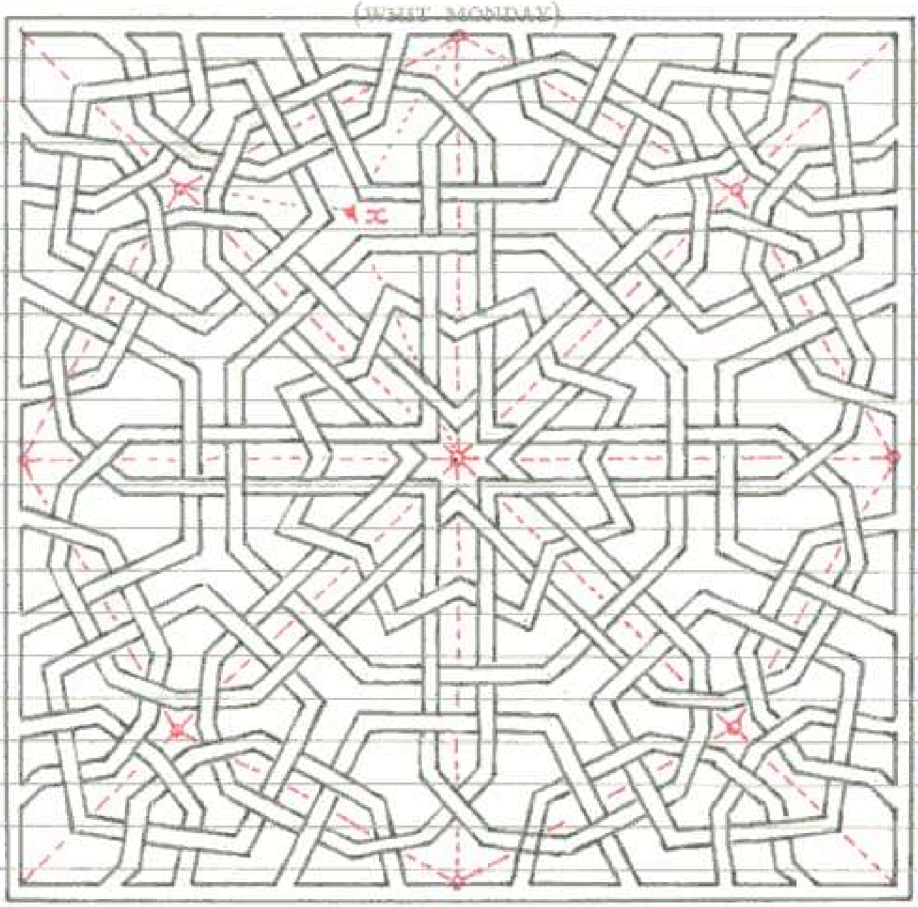
(WHIT MONDAY)

Sultan Han, Kayseri, Turkey - entrance portal.

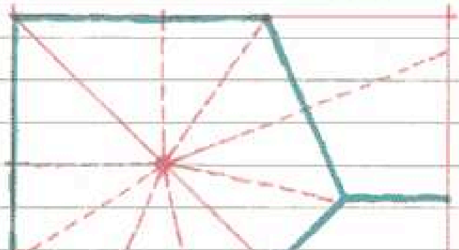
see K. Erdmann & H. Erdmann (1976) Plate 81.

This pattern is visible in Hill & Grabar (1964) fig. 471 but few details are discernible.

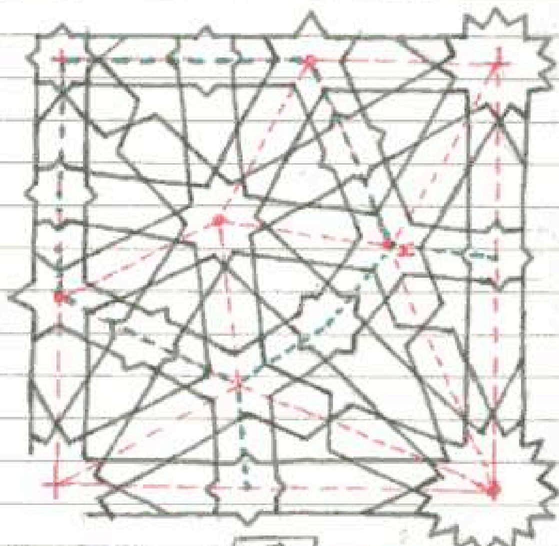
This may be regarded as a K_1K_2 pattern see p. 292 a[...] cf fig. C on p. 94.



A



B



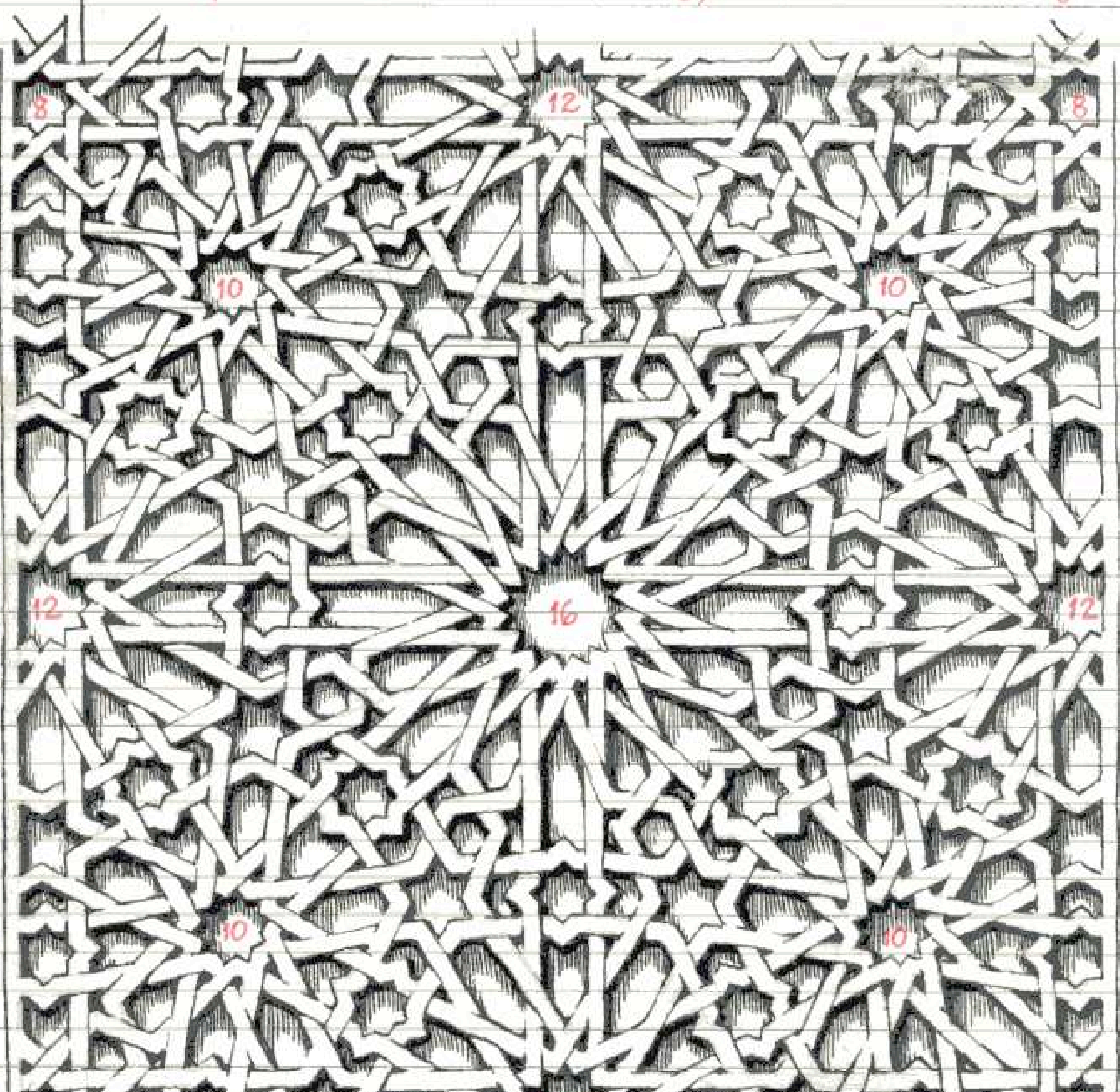
C

Sultan Han, Konya/Aksaray Ro[ad] Main Portal.

Fri 4 May 1984

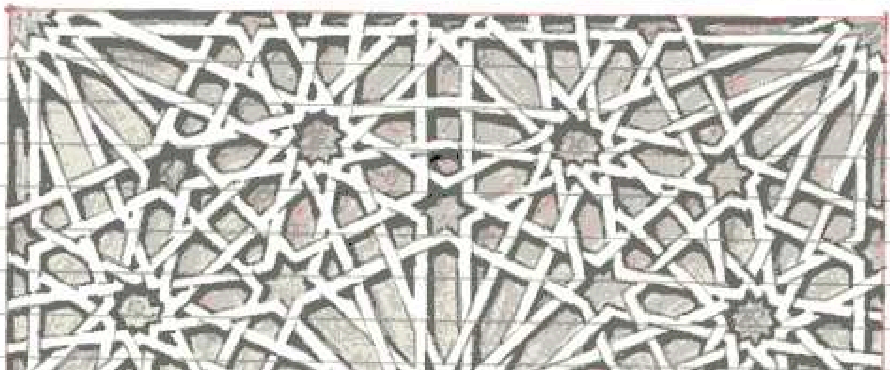
16-rayed centres, the remaining tetrad becomes an 8-pointed star and the sites of the original 3-way unions(x) on the underlying net now become 6-pointed stars. The small squares in fig. 124A are converted to khatems. A sketch of the definitive pattern (slightly modified in detail) is shown below. A similar pattern may be based on the net of fig. 122A,

Main façade of Sultan Han, Akserai (Turkey) 1229-79 A.D. (slightly modified)



The pattern opposite belongs with the K_1 & K_2 patterns. The kites are:
K_1 [4 x 6 x 4] 8, $\tilde{10}$, 12
K_2 [8 x 4 x 8] $\tilde{10}$, 12, 16

A



This belongs with the kite & rhomb patterns see fig. A, p. 96

$L8_4$ $M\tilde{10}_6$ $N16_4$

One can, of course, treat the 6-stars as main motifs, and the pattern then becomes $K_1K_2K_3$ R but the 6-stars are very irregular so this is not particularly useful.

Original pattern, base on net of fig. 122 A. cf. p. 138 →

as shown in fig. 126 above. However, this does not appear to exist as an authentic pattern in this exact form, but a geometrically similar pattern occurs in ~~Morocco~~ Algeria, as pierced grilles above the mihrab of the Sidi Belhasan mosque, Tlemcen, where the 8-, 10- and 16-centres are treated as rosettes.* see p. 138

A further sequence of variation may be said to start with the discovery mentioned on p. 121, illustrated in fig. 127A. As previously noted, this net cannot be achieved exactly. The notion of a doubled hexagon between the octagonal centres may however have arisen from an observation of this kind. Fig. 127B shows the application of this notion in the formation of a new net; here the octagons are regular, and regularizing the hexagons allows the pentagons to be made almost regular in turn. It is then found that hexagons (b) and pentagons (c) now line up with

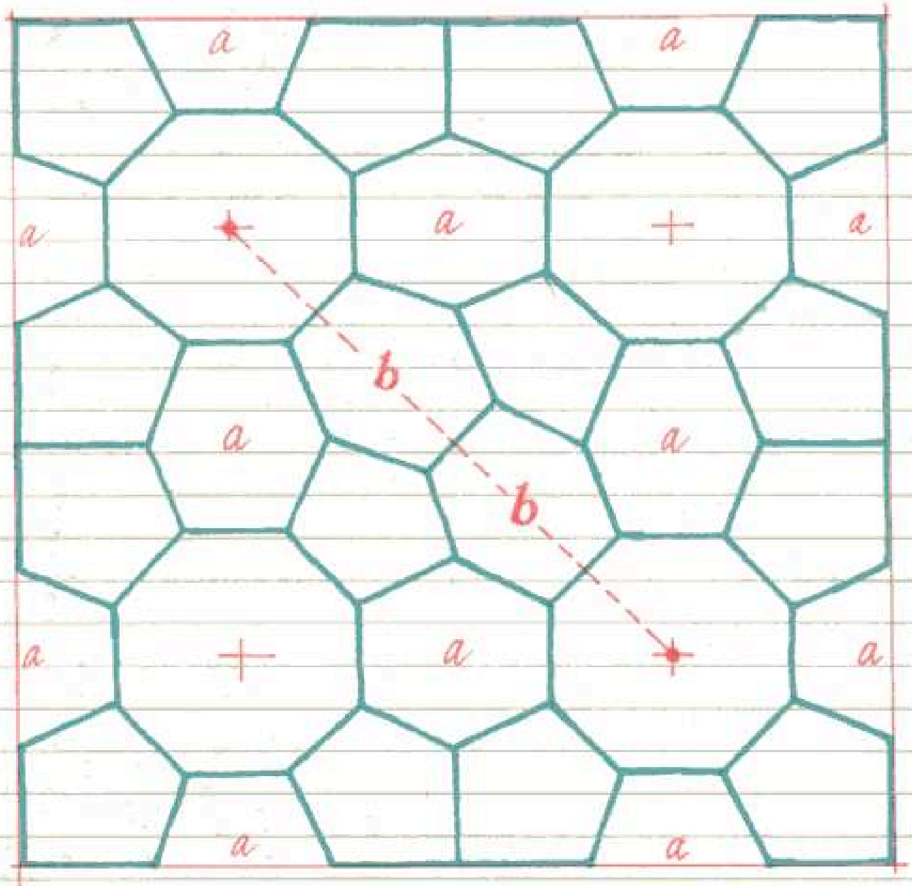
* It may be possible to adapt this pattern to the grid of fig. 122B.

127

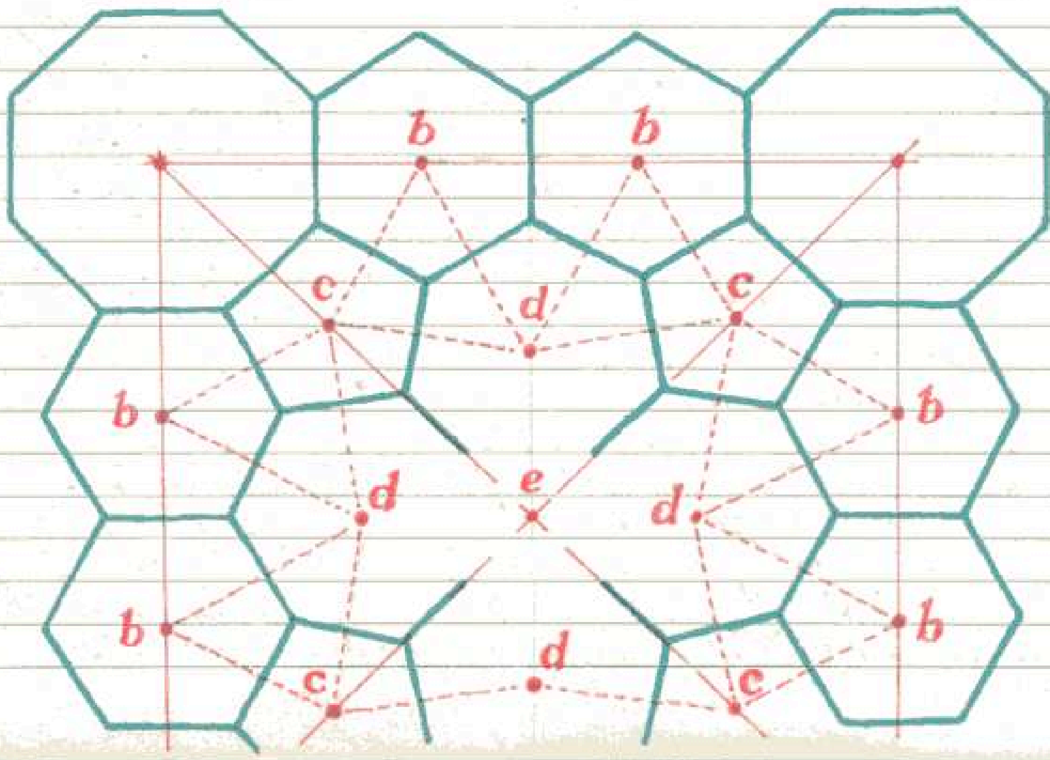
Fri 11 May 1984

Thursday, JUNE 2, 1966

5,6,8 net with diagonal double-6



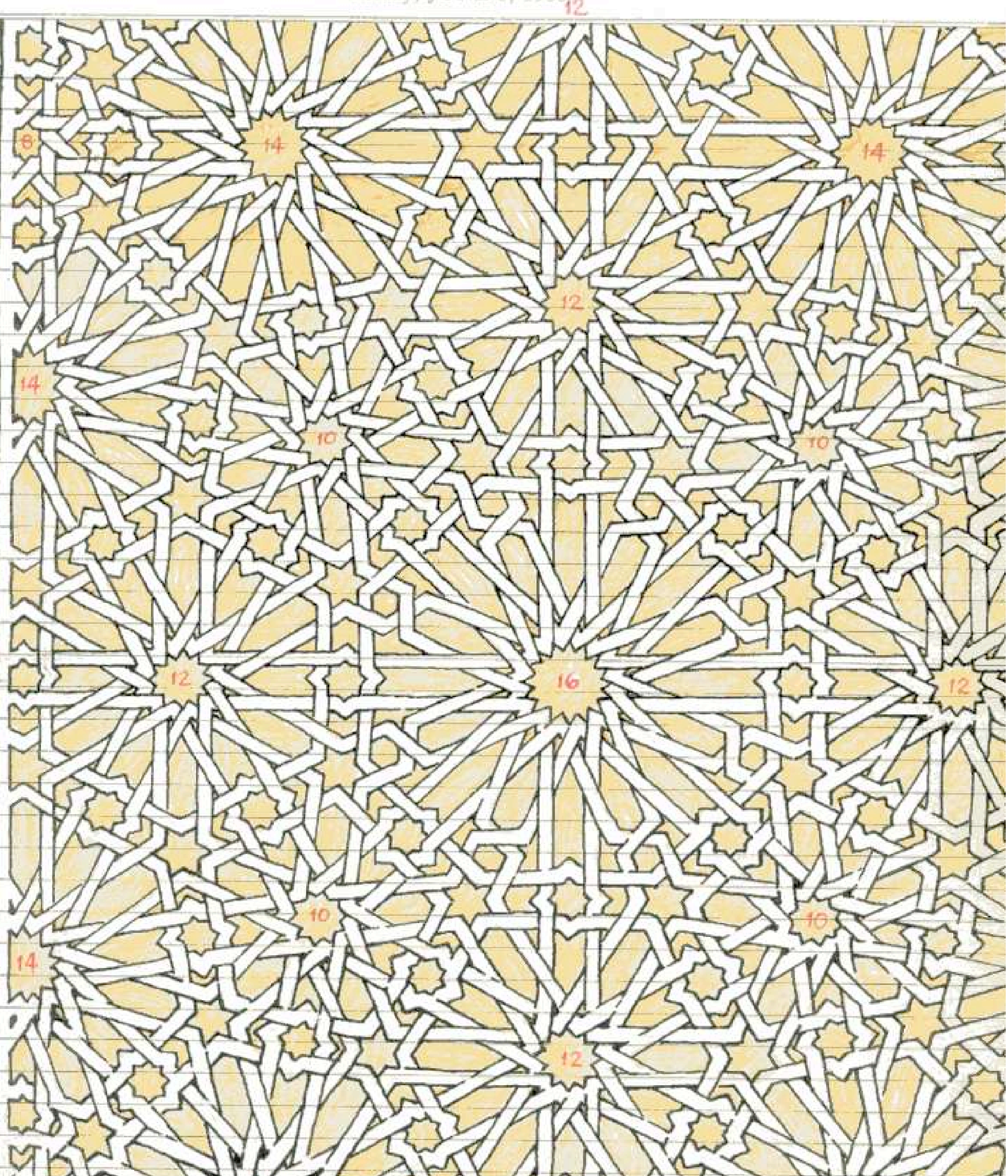
A



B

Tue 8 May 1984

Friday, JUNE 3, 1966



Original pattern, using 6, 8, 10, 12, 14 and 16-pointed stars.

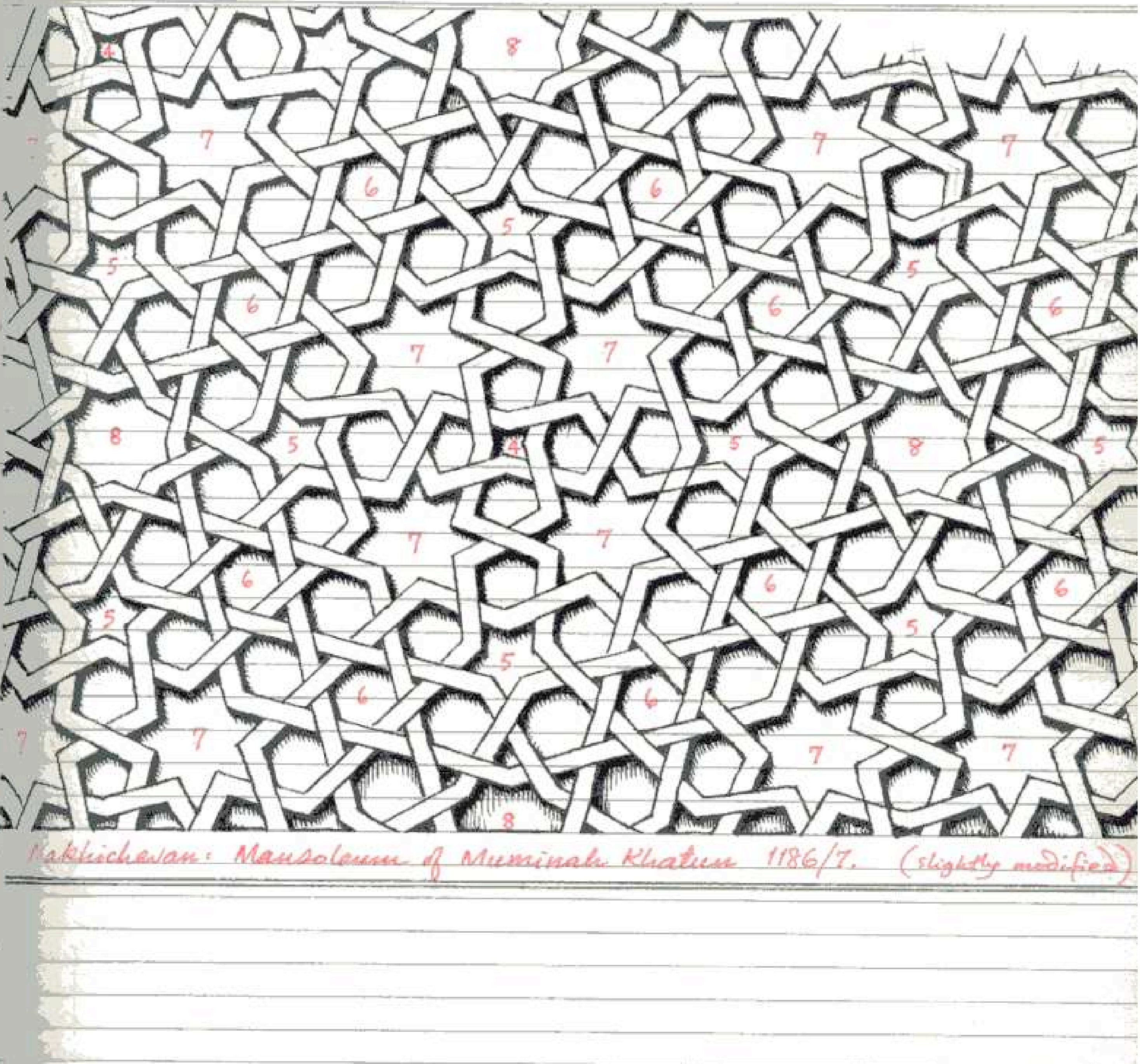
Sat/11 May 1984

Saturday, JUNE 4, 1966

new centres (d), the angles at which are close enough to heptagonal angles to enable nearly regular heptagons to be drawn, leaving a small square at (e). We thus have a "pseudo-regular" tessellation of octagons, heptagons, hexagons, pentagons and squares. A number of patterns are extant in authentic Islamic ornament: that shown opposite is from the 12th century mausoleum of Mu'minah Khatun at Nakhichevan. Bourgoin (1879) has another, Plate 163, probably from the mosque of Sarghatmish in Cairo, and the earliest of these examples seems to be a version from the mihrab of the mosque at Barsian, Isfahan (1134 A.D.). In fact, many patterns which have as their basis the green net on fig. 122B can be adapted to this new net (fig. 127B). One such adaptation is illustrated on p. 128 which adapts the pattern of p. 125 to the new net of p. 127B.

It is of great interest, and may well be significant, that both the "double-six" (fig. 130) and "single-six" versions (fig. 123) occur on the Mu'minah Khatun mausoleum. These treat the motifs as simple stars; Bourgoin's version (1879, Pl. 163) ~~and the Barsian pattern~~ treats them as ~~stars~~ rosettes. Many other varieties are possible, as already mentioned.

Sunday, JUNE 5, 1966



Nakhichewan: Mausoleum of Muminah Khatun 1186/7. (slightly modified)

Tue 15 May 1984.

Patterns with generalized rosette outer cells

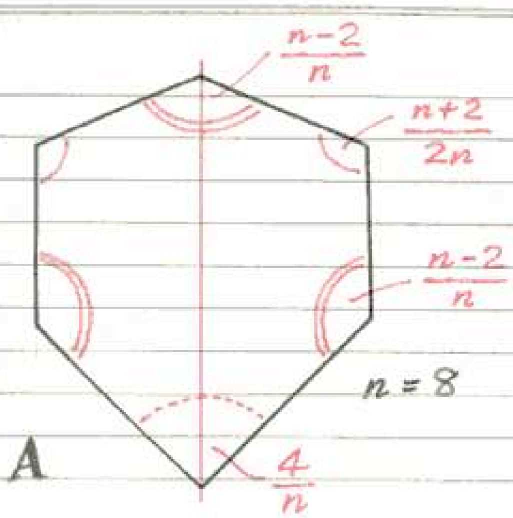
If we take the green hexagons of fig. 122B we can generalize this shape (fig. 132C) and form some interesting patterns from it. We generalize the hexagon to the extent that base angle a can have any value, i.e. n does not need to be an integer. The generalized hexagon we are here concerned with ~~the~~ is equilateral with opposite sides parallel. The interior star is inscribed on the midpoints of the edges of the hexagon and certain edges of this star are parallel to edges of the hexagon, where noted on fig. 132C. The angles of the points of the interior star are all equal, and their bisectors are perpendicular to and bisect the edges of the hexagon. Any tiling using these hexagons will thus generate an arrangement consisting mainly or entirely of parallel-sided rosette cells (fig. 132 A, B) in which terminal and subterminal segments are equal. Since these outer cells do not form rosettes (fig. 112 A) the value of n does not have to be integral. If parallel sided cells are formed, points (a) - fig. 132C - coincide when $n = 10$, and overlap when $n > 10$. However, the parallelisms noted in fig. 132C need not be observed, provided that the points of the interior stars of the hexagons are all equal and that the bisectors of these angles form perpendicular bisectors of the sides of the hexagons.

In the arrangement of fig. 132D, if the outer cells are parallel-sided (i.e. the parallelisms of fig. C are observed) then the central p-gon can have an inscribed (p/2)1-star. In this latter radial pattern the angular values of the hexagon are precisely determined, but the angles of the inscribed stars can, of course, be varied infinitely. The essential data for constructing any of these radial patterns are given in the table opposite - fig. 132F. I know of no authentic Islamic ornament which uses such arrangements however.

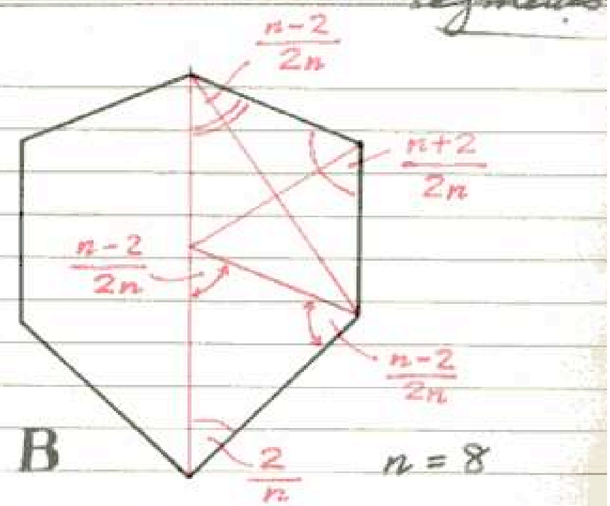
There are two principal ways of arranging the symmetrical hexagons of fig. 132C to form periodic patterns, illustrated in fig. 136A, B. Patterns of rosette outer cells based on these are not common, but those derived from pattern A probably

Tue 15 May 1984

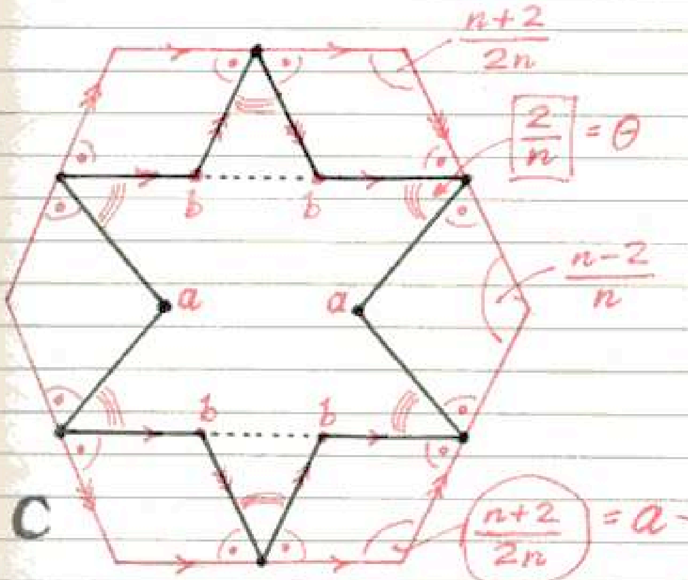
Hexagons generating parallel-sided ontes cells – with equal terminal & subterminal segments



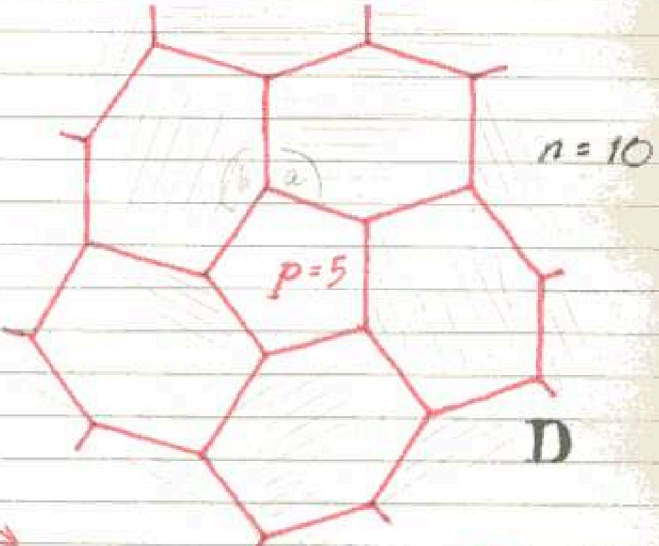
A



B



C



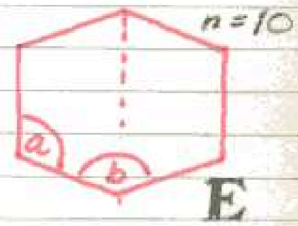
D

If hexagons must fit round a central p-gon, then $a + b = \frac{p+2}{p}$

i.e. $\frac{3n-2}{2n} = \frac{p+2}{p}$

from which $n = \frac{2p}{p-4}$

Some suitable values are shown on the left.



E

p	n	$a°$
3	−6 *	60°
4	∞ †	90°
5	10	108°
6	6	120°
7	4.67	128.57°
8	4	135°
10	3.33	144°
12	3	150°

F

* 'hexagons' are two equilateral triangles in 'bow-tie' formation

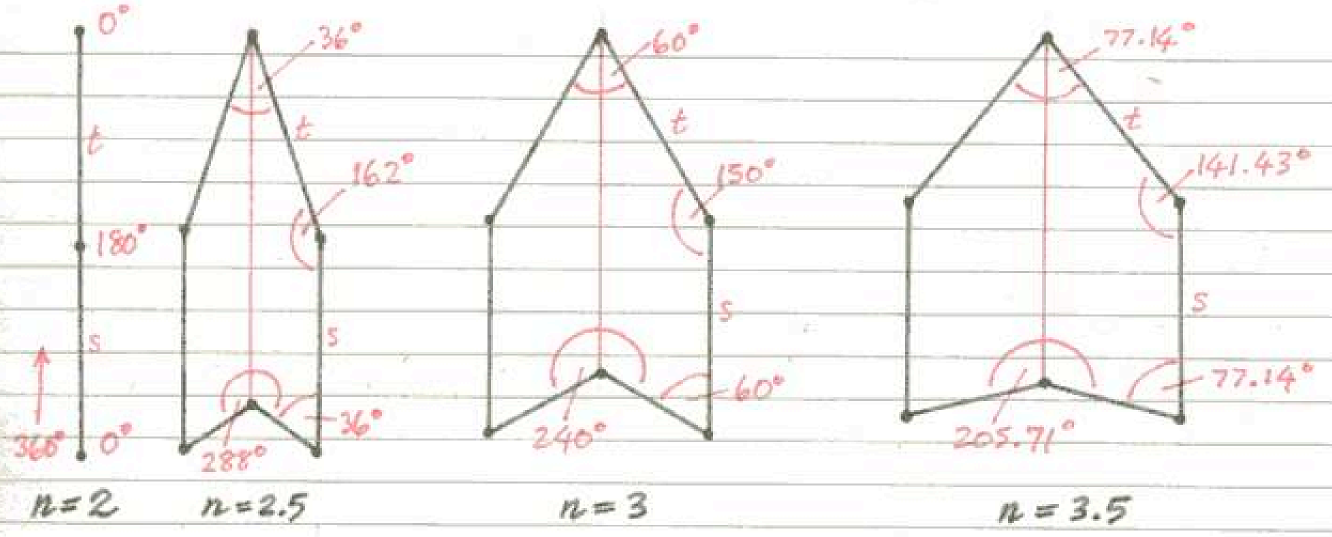
† 'hexagons' are 2×1 rectangles

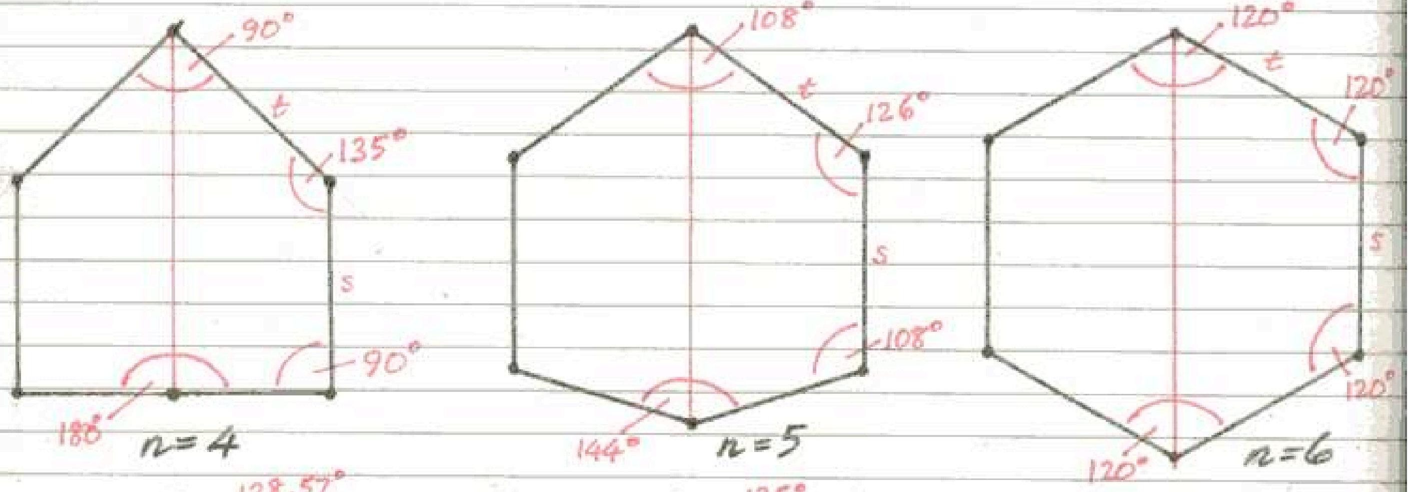
133 | Rosette Outer Cells

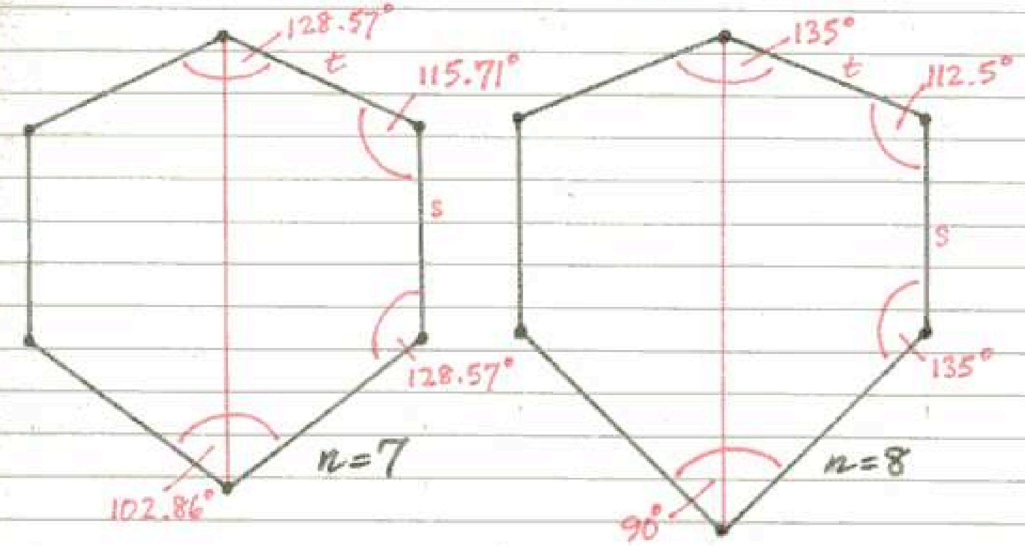
for different values of $\underline{n}$.

Thursday, JUNE 9, 1966

After Wed 16 May 1984





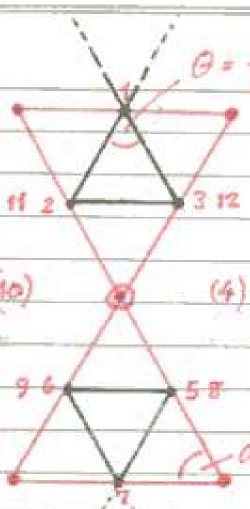


Outer cells of different sizes of rosettes — all parallel sided, with equal terminal and subterminal segments. For various low values of $\underline{n}$, the rosette number.

t = terminal segment

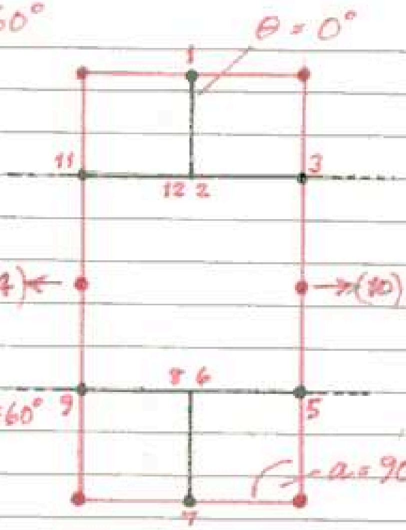
s = subterminal segment

When $p=3$ points 4, 10 are imaginary.



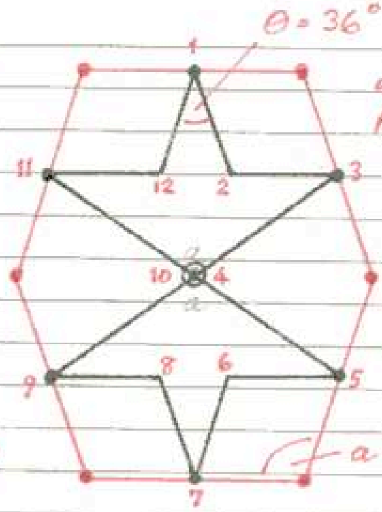
$n = -6$

$p = 3$



$n = \infty$

$p = 4$

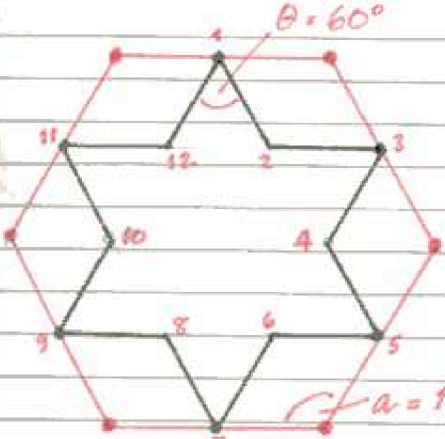


$n = 10$

$p = 5$

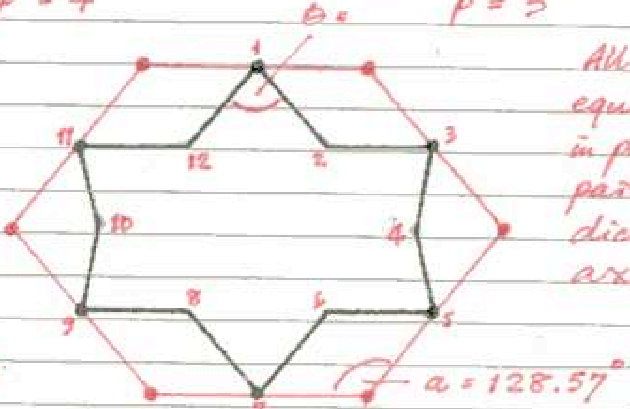
When $n > 10$ points 4 & 10 overlap.

When $p = 4$ points 4, 10 are at infini[ty] ($n = \infty$)



$n = 6$

$p = 6$

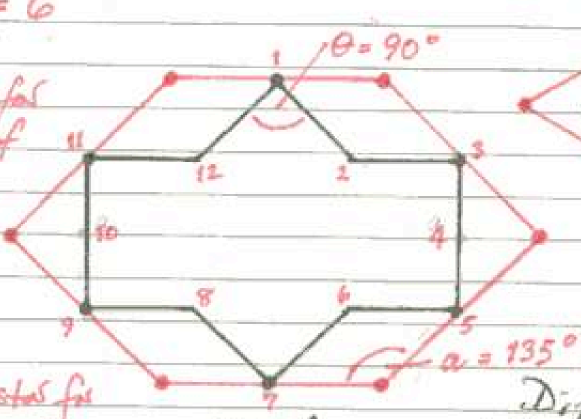


$n = 4.67$

$p = 7$

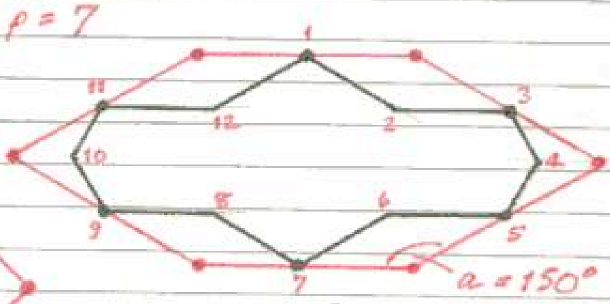
All hexagons are equilateral; sides in parallel opposite pairs; two perpendicular mirror axes.

See fig. 132D, E for the relevance of the p-values.



$n = 4$

$p = 8$



$n = 3$

$p = 12$

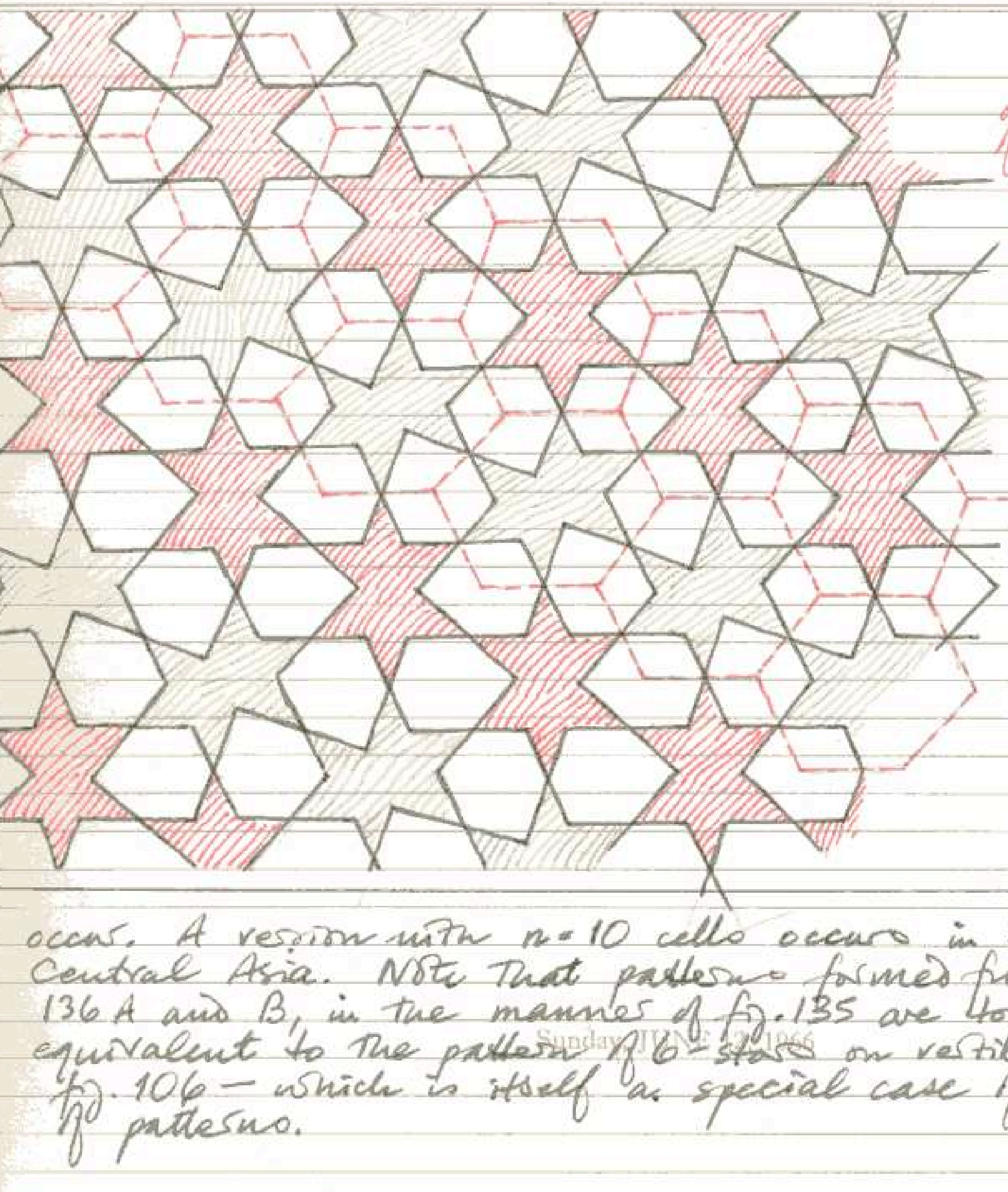
N.B. The inscribed star for $p = 10$, $n = 3.33$ is often used in dodecagonal patterns of type I. See for example, Bourgoin (1879) Pl. 177, - although his version is very inaccurately drawn.

Different shapes of hexagons taken from values given in table 132F.

135

Wed 16 May 1984

Saturday, JUNE 11, 1966



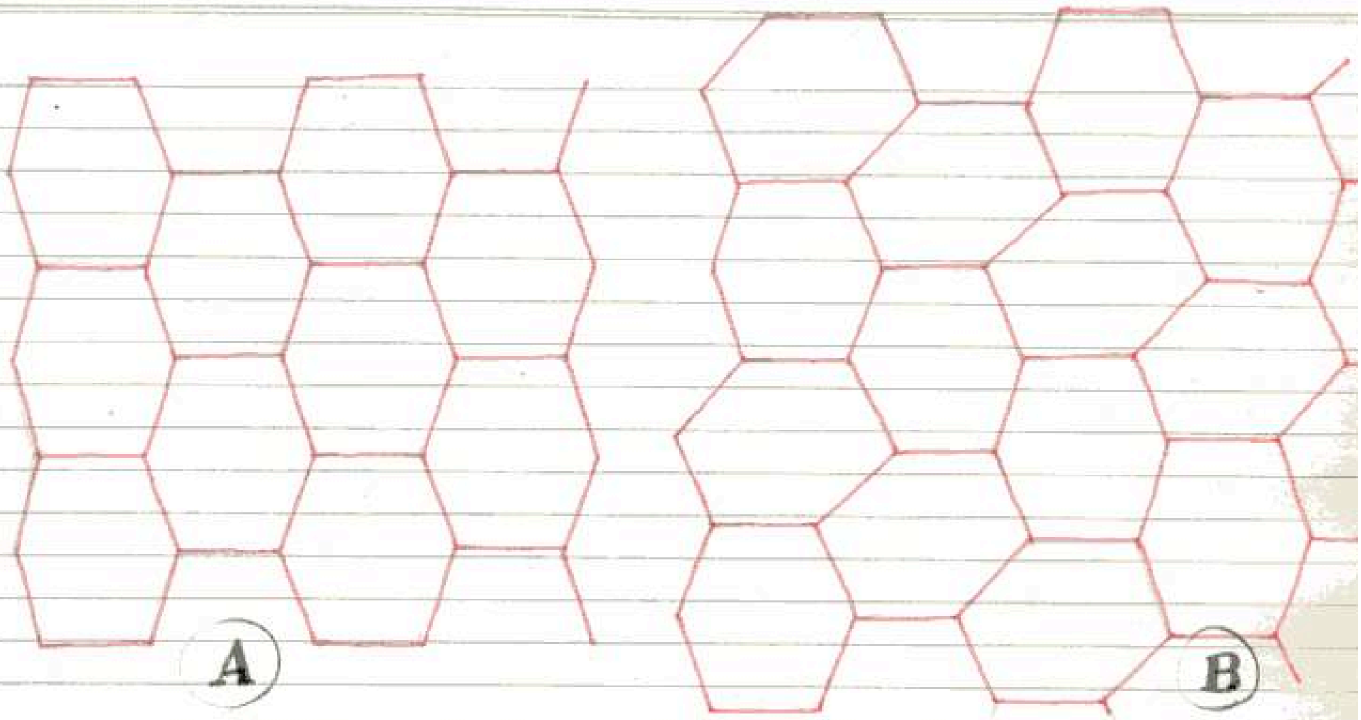
A pattern of rosette cells on the basis of fig. 136 B, opposite. Zones of similarly orientated hexagons and inscribed stars are similarly coloured. Patterns of this kind can be composed of cells relating to any size of rosette. Indeed, the cells need not even relate to an integral rosette

Mon 14 May 1984
(original)

occurs. A version with $n = 10$ cells occurs in India and Central Asia. Note that patterns formed from tilings 136 A and B, in the manner of fig. 135 are topologically equivalent to the pattern of 6-stars on vertices of $\{3,6\}$ – fig. 106 – which is itself a special case of this group of patterns.

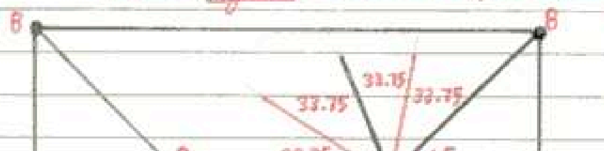
Sunday, JUNE 12, 1966

Monday, JUNE 13, 1966

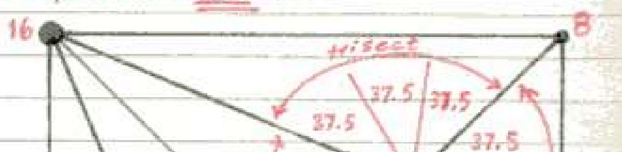


The two principal ways of forming periodic tilings with the symmetrical hexagons described on p. 131 and fig. 132C. An example of a pattern using tiling B is shown opposite, on p. 135.

For a regular 10-centre, each angle is equal to 36°:—



C see p. 137

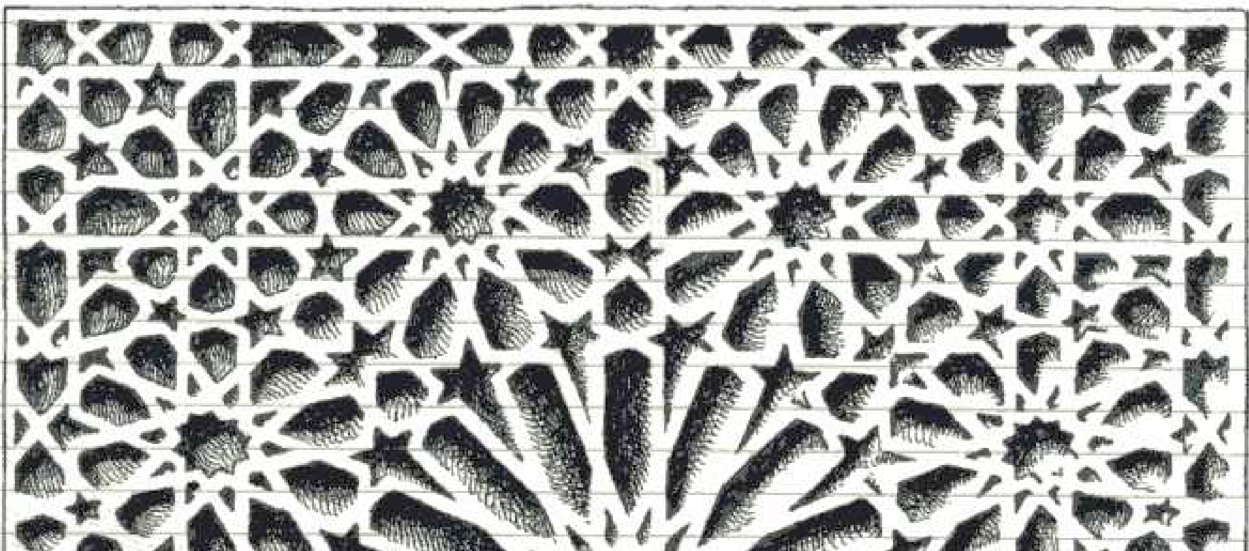


D see p. 138 also p. 126

Construction of main radii of the approximately regular 10-centres in the patterns overpage, on pp. 137 and 138 respectively.

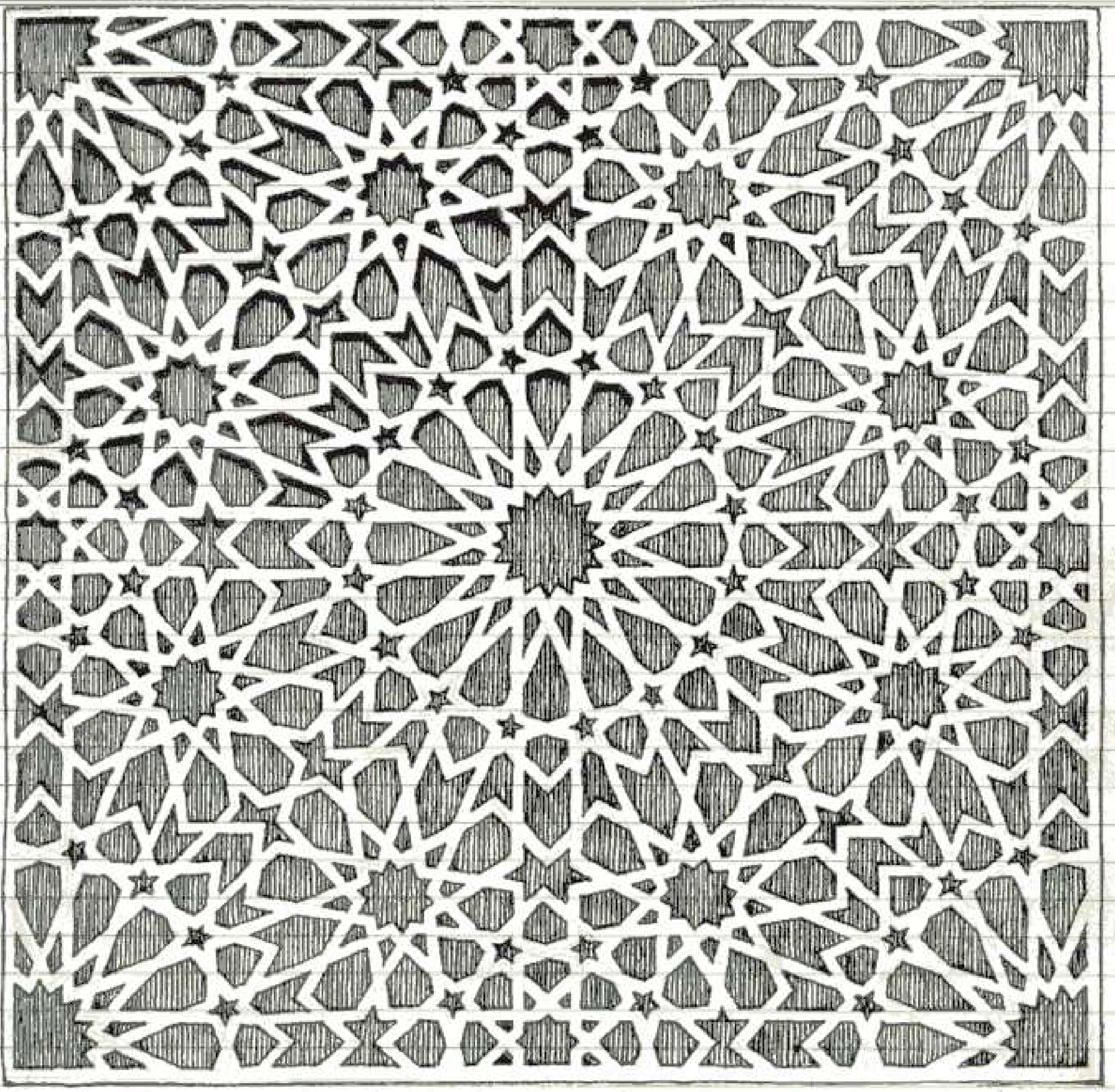
→

Thu 17 May 1984



Stucco, Morocco.

The underlying frameworks of these two patterns (p. 136C, D) show that the location of the 10-centres is identical in each case, but the construction of the main radii of these approximately regular centres differs in each case. Of the two, the Sidi bel-Hasan grille gives the more accurate result, but neither allows complete motifs to be realized at the 10-centres. In the stucco



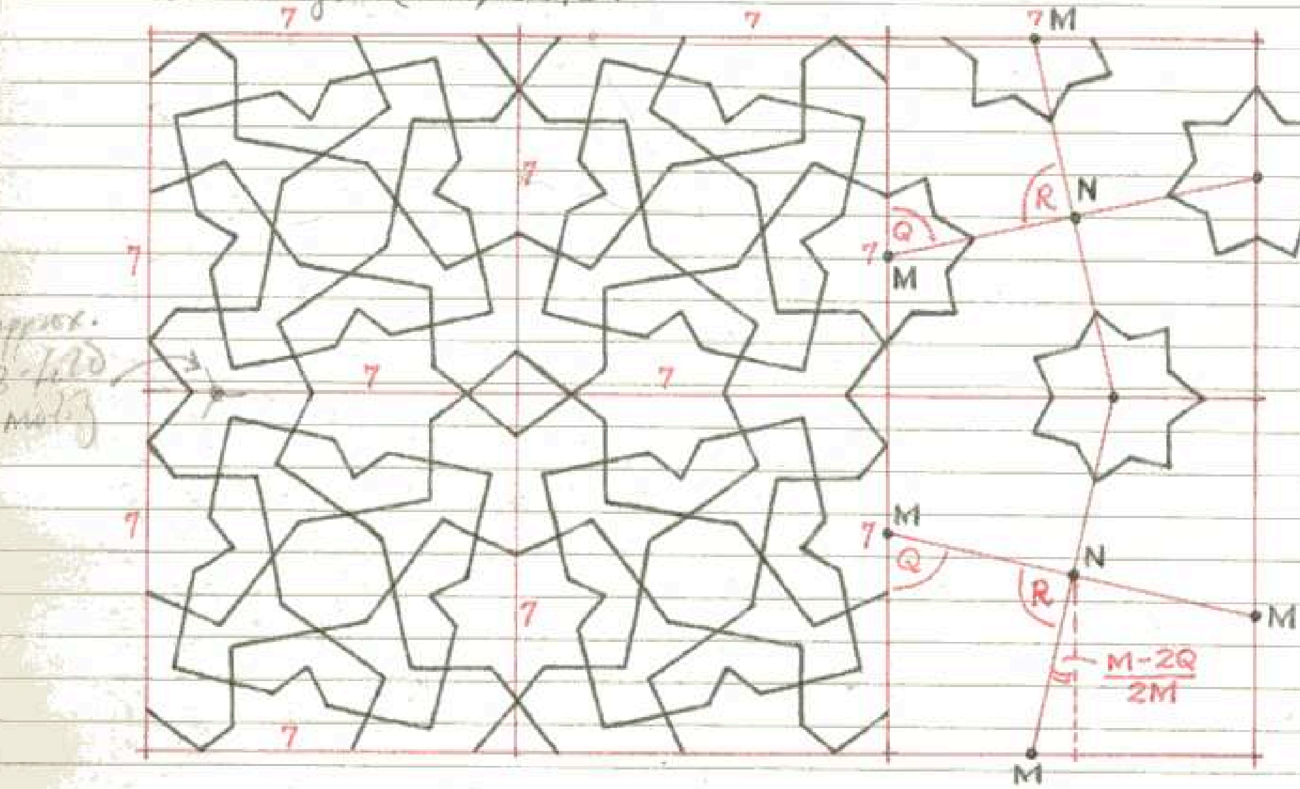
Pierced grille above mihrab of Sidi bel-Hasan mosque, Tlemcen, Algeria. 1296 A.D.
Proportions are approximately those of the original. Compare fig. 126.

pattern the 10-fold "rosettes" are decidedly egg-shaped and in fact the method of construction gives oval central stars unless means are taken to correct this (both features are present on the original stucco panel from which this pattern is copied, and the original is far less regularly drawn than our drawing, in many of its details). Both patterns require a great deal of freehand work for their completion, rather than exact geometry.

139 [illegible] Fri 18 May 1984

Thursday, JUNE 16, 1966

From Bourgoin (1879) Pl. 170 :-



This is representative of a group of patterns constructed within squares, and related to the Kite + rhomb group of pp. 97 and 98. There is no motif at centre L in the above group, although often a pseudomotif is situated at this point, as in the above pattern.* (Bourgoin classifies the pattern above among his "heptagonal" group). The following arrangements on this basis have been discovered:

M	N	Q	R	$\frac{M-2Q}{2M}$	
7	{8}	3	4	12.86°	Bourgoin (1879) Plate 170. ←
3	4	1	2	30°	[illegible] Feb 1965
5	4	2	2	18°	[illegible] 21 Feb 1965
9	{8}	4	4	10°	[illegible] 14 Feb 1965
11	{8}	5	4	8.18°	[illegible] 15 Feb 1965
13	{8}	5	4	20.77°	[illegible] 16 Feb 1965

This pattern is fairly widespread in Middle East and Central Asia. It also occurs transposed to other arrangements.

See overpage →

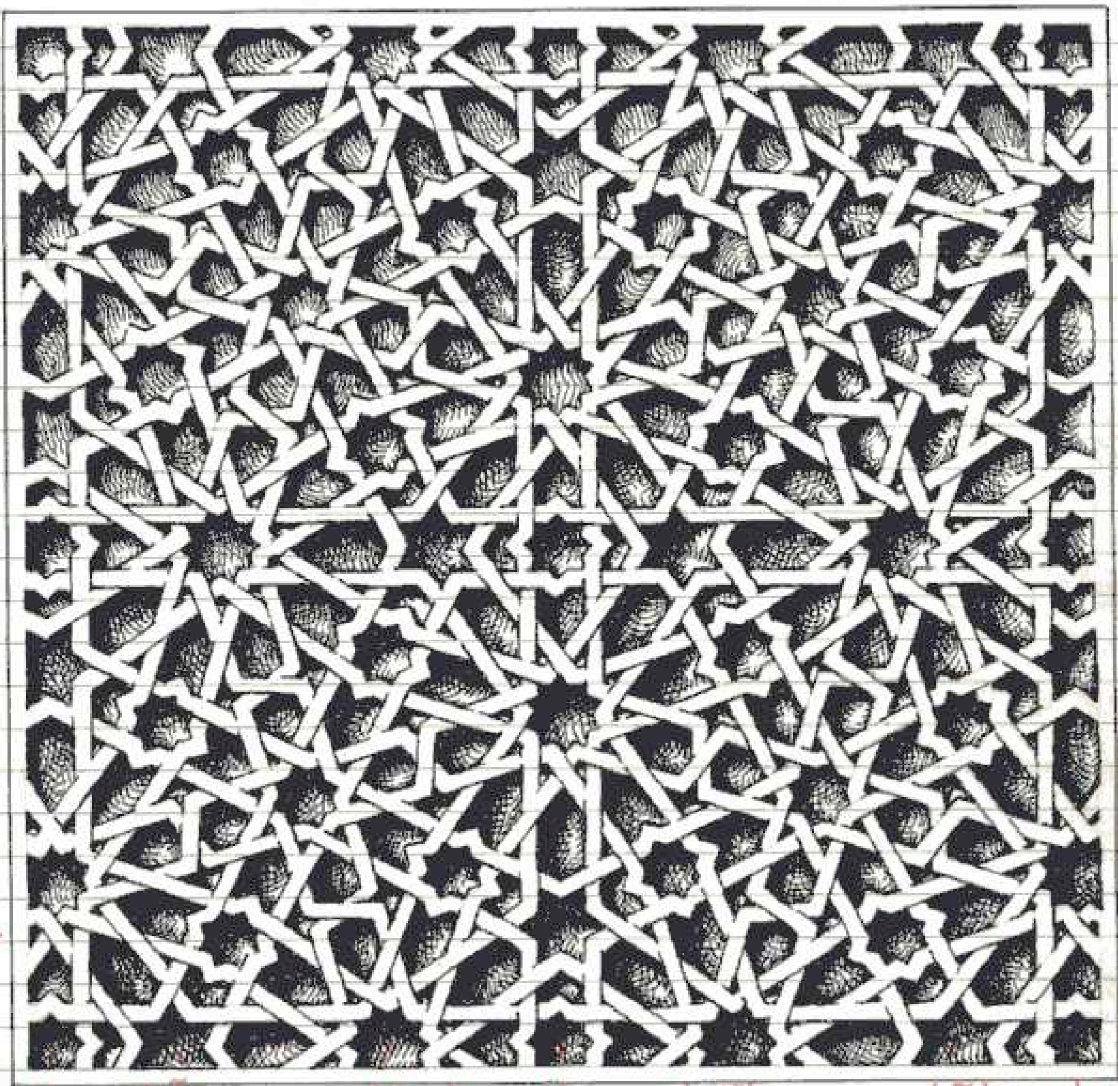
* If these are considered as 3-fold motifs then the pattern belongs with the Kite + rhomb patterns, pp. 97, 98 with "semi-symmetrical" Kites: L=3, P=2; M=7, Q=3.

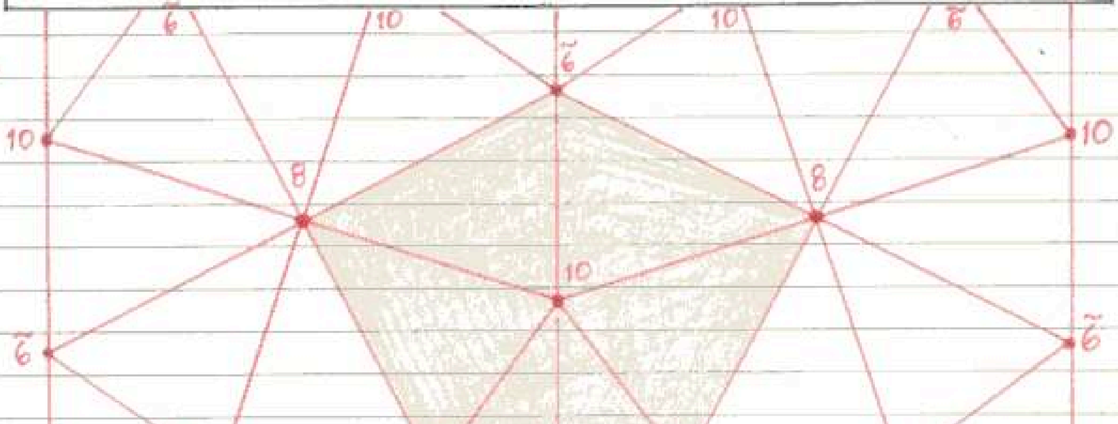
original pattern
24 March 1976

The pattern on the right belongs to the R?te + rhomb group of pp. 97, 98.

The 8- and 10-centres are drawn "regularly", the 6-centres are almost regular.

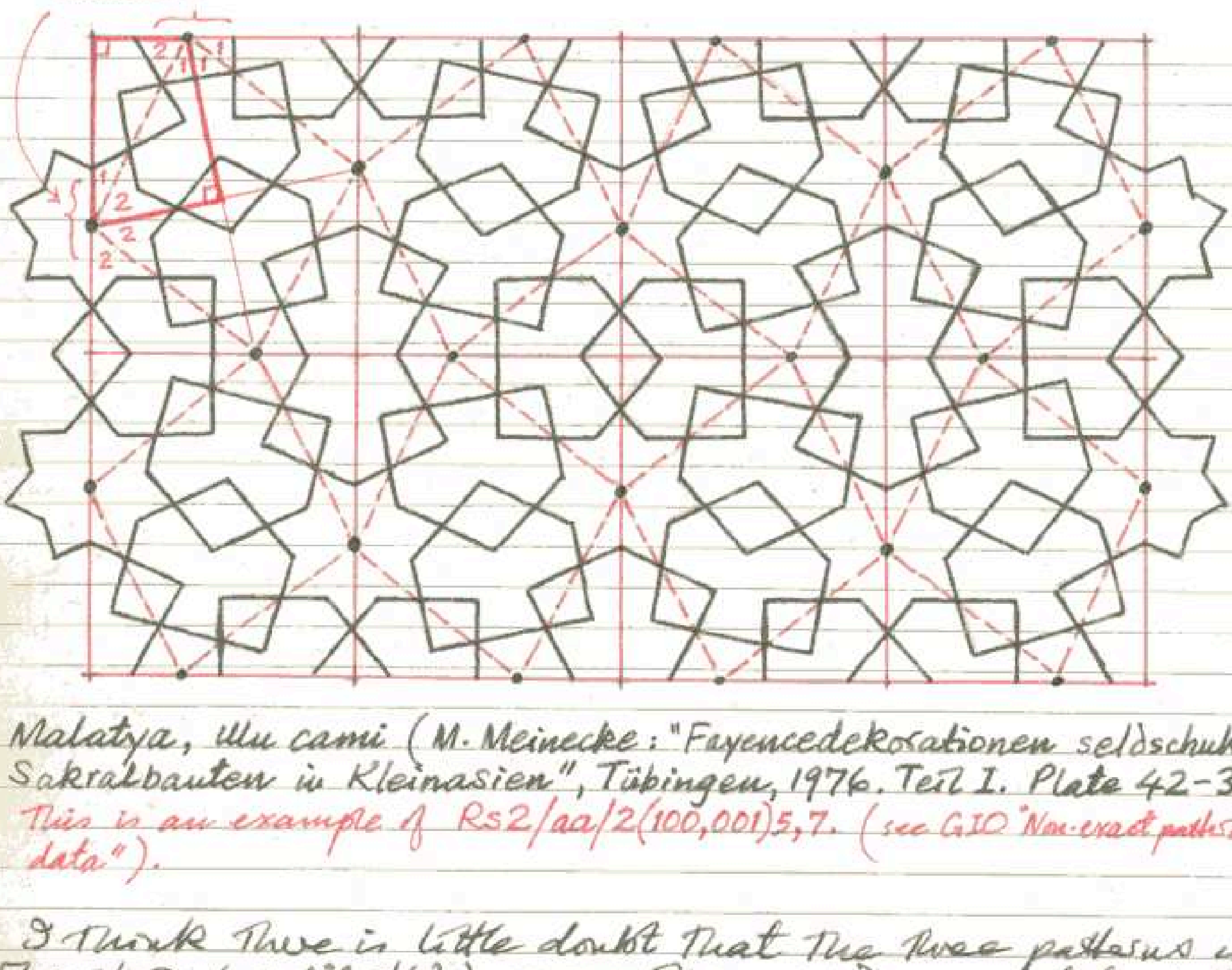
The style of realization is clearly that of the Sultan Han, Aksarai pattern shown on p. 125. This pattern can therefore be considered as derived from a pattern of non-regular pent-agons, related to the dual tiling to $5\left[\begin{smallmatrix}4\\4\end{smallmatrix}\right]$.





Afhx Fri 18 May 1984

Partition index below is differences between these numbers

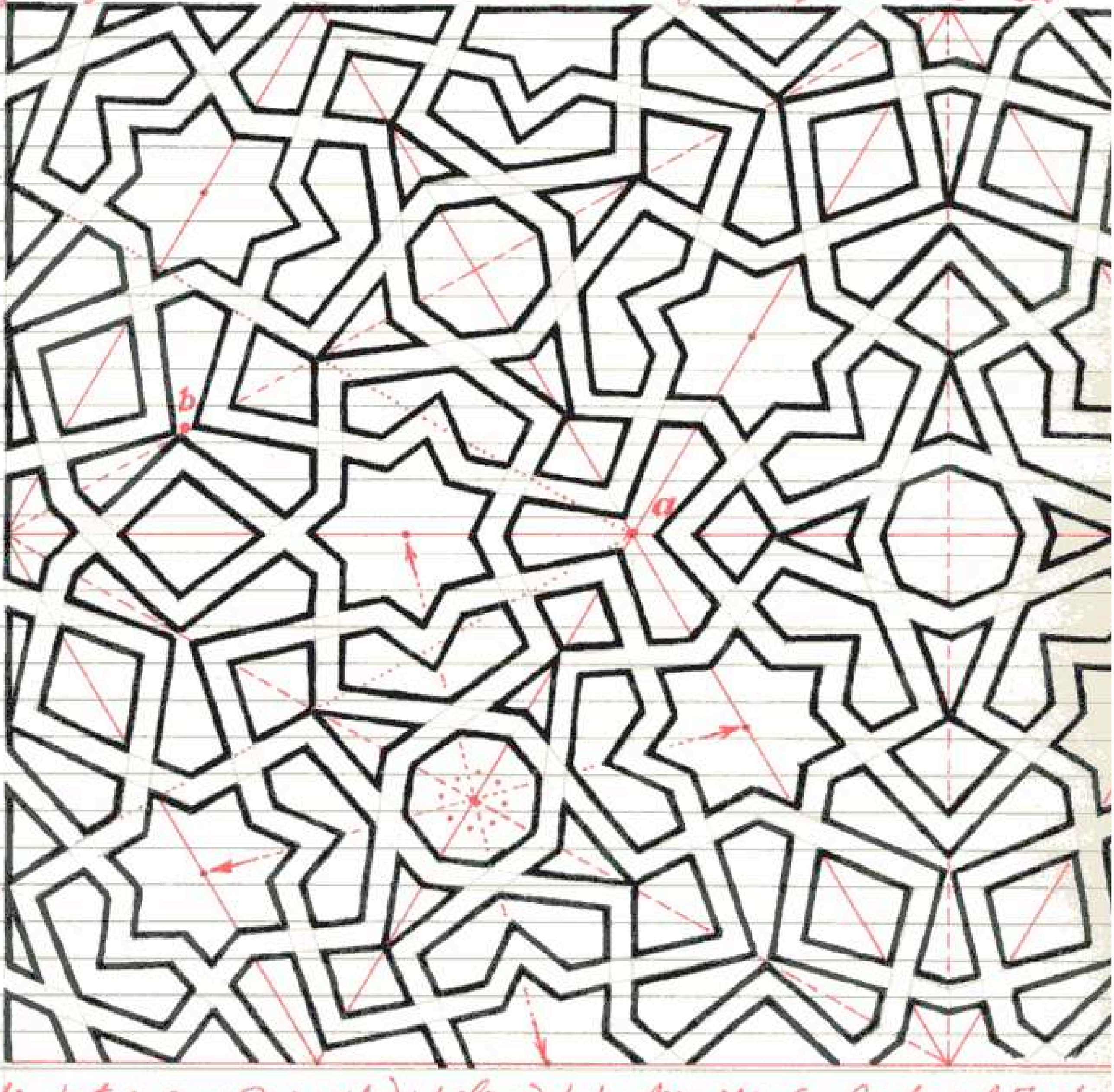


Malatya, Ulu cami (M. Meinecke: "Fayencedekorationen seldschukischer Sakralbauten in Kleinasien", Tübingen, 1976. Teil I. Plate 42-3).
This is an example of Rs2/aa/2(100,001)5,7. (see GIO "Non-exact patterns. Basic data").

I think there is little doubt that the three patterns with 7-stars (pp. 139-142) were not composed completely independently of one another, but that there was a conscious effort on the part of various pattern designers to adapt a certain motif style to different underlying bases. The pattern of 7-stars above, on an Rs2 basis, is known in other styles e.g. with rosettes, in Asia minor, but patterns with 7-stars as the main motifs are rather rare on the whole.

Fri 18 May 1984

Based on $\{6,3\}$. Octagons at mid-edges. 6-stars at face centres. 7-stars on main radii of hexagons. 7-stars are therefore on vertices of semiregular tiling $r\{{3 \atop 6}\}$.



Authentic, but source not recorded; believed to be Asia Minor. Centres of 7-stars determined by radii from centre of the <u>regular</u> octagons. 7-stars <u>regular</u>.
A camera lucida tracing from a photograph is available. In the original pattern some attempt has been made to draw the false triads (b) to look more like the true triad figures (a). The 6-stars are also somewhat larger than shown in the above construction.

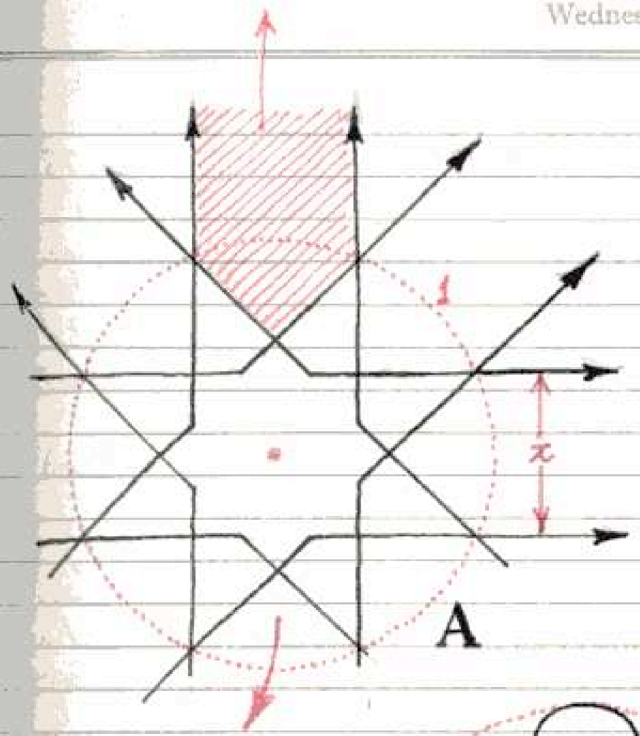
 After Mon 21 May 1984

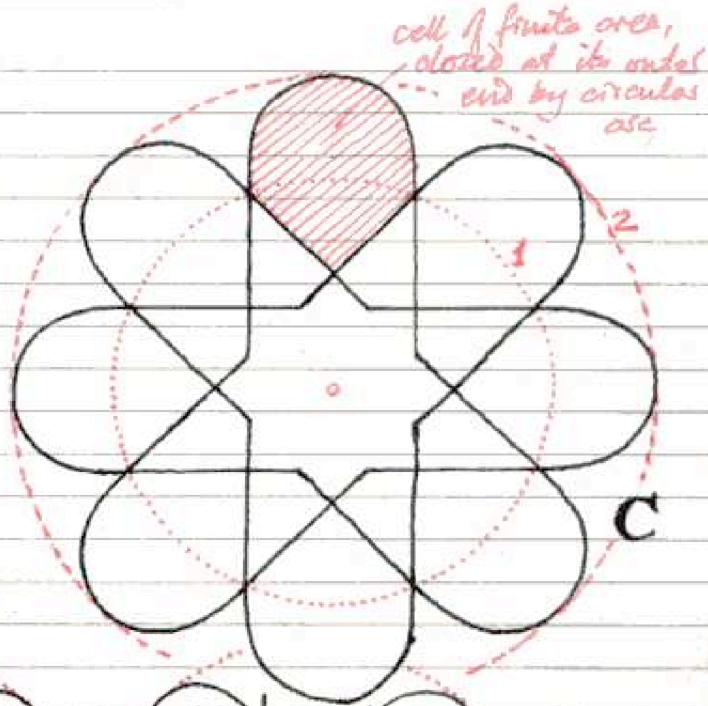
The Historical Origin of the Geometrical Rosette

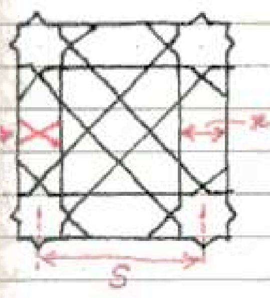
Although it is easy to derive the generalized n-rayed rosette from the 6-fold prototype on a formal, geometrical basis, this may not have been the actual, historical method of discovery of the general n-fold rosette. As shown above (pp. 9, 10; 103 et seq.) the formal derivation produces parallel sided rosettes with collinear terminal segments, in which terminal and subterminal segments are of equal length. Outer cells are always symmetrical hexagons, but only when n = 6 are they regular polygons.

In an (8/3)2 star (fig. 144 A) if we extend the sides, pairs of parallel sides enclose between them infinitely long areas (shaded in the figure), which can be closed by any kind of capping device, as in fig. C, to produce a kind of rosette like motif with finite outer cells. Here the cap is formed by a circular arc. These motifs can be arranged in a repeating pattern by placing them, tangent to one another, on the vertices of a square repeating unit (ABCD, fig. 144D). Initially the ratio between the circumcircles of the (8/3)2 star (1, fig. C) and that of the rosette motif (2) would be arbitrarily chosen, but it would be natural to complete the space between the four motifs by a small circle centred on point a, tangent to the four surrounding motifs. This procedure is seen to give rise to 5-cusped curvilinear "peripheral stars", and a first stage towards trying to regularize these might have suggested itself, i.e., to circumscribe it with a circle centred on b, passing through points c and d. This would ensure that all points of the peripheral star were equidistant from point b, and it would also fix the ratio between radii be and De, and also in addition the ratio between width x and the side s of the square repeating unit, ABCD. A subsequent modification produced by joining points such as c & d would yield the typical geometrical rosettes (fig. 108 B) with regular octagonal interstitial cell in place of circle on point a.

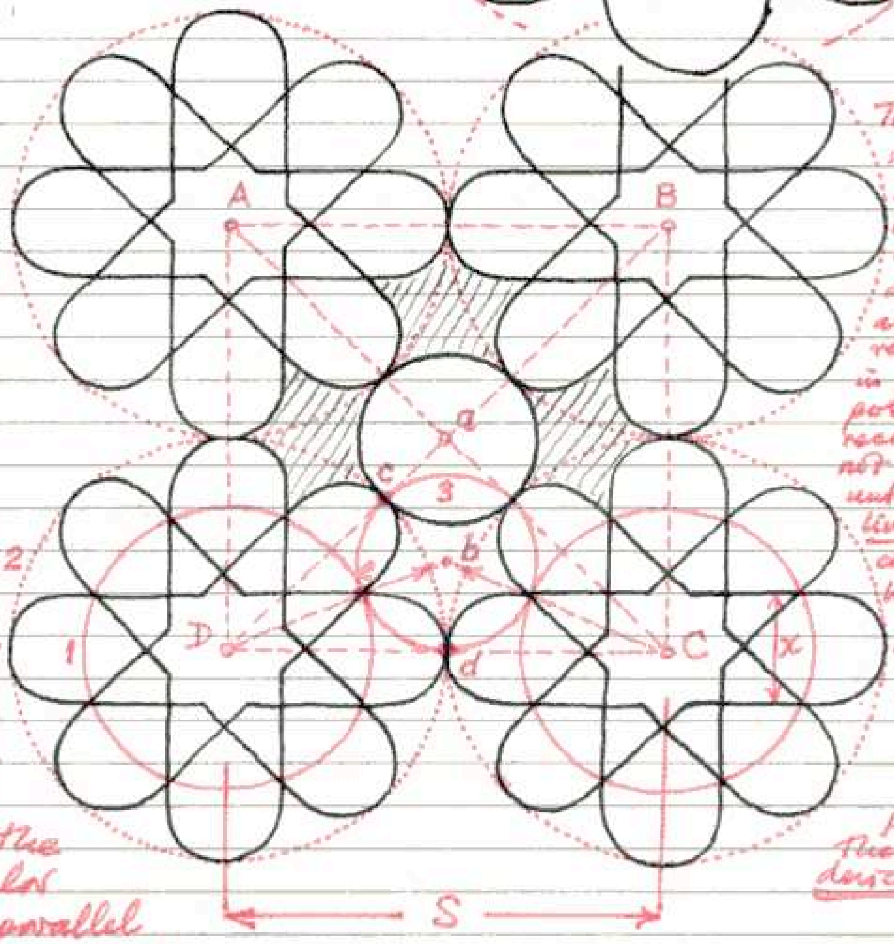
Wednesday, JUNE 22, 1966







B



Initially, ratio of radii of circles 1 & 2 is arbitrary, but regularization of "peripheral stars" by circle 3 on point b fixes size of circle 1 and leads the way to the formation of regular 8-rosettes with parallel sides and equal terminal and subterminal segments, and collinear terminal segments.

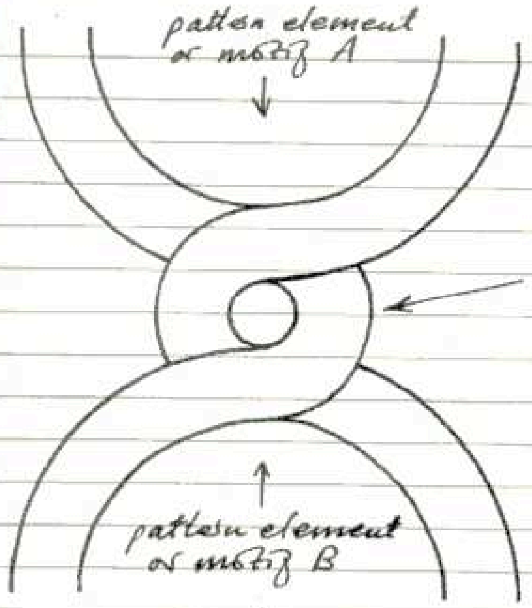
The construct shown, where circle 3 deter mining size of circle 1 depends on ability to recognize "sta in peripheral positions. This recognition may not have occurred until after the linear form of capping device had evolved, perhaps from the pattern in fig. B. (? – This depends on the prevalence of the capping device at the time

Mon 21 May 1984

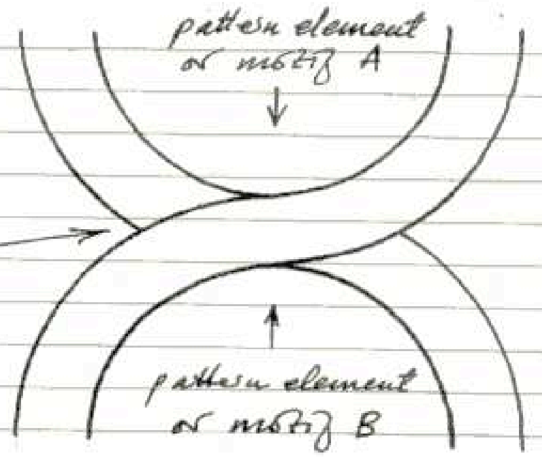
After Mon 21 May 1984

Rosette-like motifs such as that in fig. 144C were known in Madinat az-Zahra. B. Pavon Maldonado (1975, "El Arte Hispano-Musulman en su Decoración Geométrica") has traced the development of many similar motifs (see pp. 127 et seq.). Clearly, there was a great fund of rosette-like motifs from a variety of sources preceding the first development of true geometrical rosettes. Many different methods of rounding off the peripheral regions of stars and star-like figures to produce pleasing motifs were used either as isolated medallions or as part of periodic patterns. Creswell (Muslim Architecture of Egypt I pp 213-215) traces the origin of one six-fold motif and subsequent use through 9 centuries - originally Syrian, passing into Byzantine and afterwards into Muslim Art. Such curvilinear motifs undoubtedly played a part in the discovery of the definitive geometrical rosette - indeed, in origin the geometrical rosette may be simply a completely rectilinear version of the kind of motif shown in fig. 144C,D — but until or unless the historical origin of the geometrical rosette can be pinpointed more accurately, in time and place, it is not worthwhile to indulge in pure speculation. The earliest 6-fold rosette seems to be shown on the tympanum of the main façade of the Arab-Ata mausoleum at Tim, in Uzbekistan (977-8 A.D.), but the configuration itself was already in existence long before this, embedded in the well known pattern of 6-stars (see drawing on p. 106), and its use as a discrete motif at Tim may have little or nothing to do with the development of the generalized n-fold rosette. One of the stucco soffits of the Ibn Tulun mosque (fig. 148A) has a pattern with strikingly rosette-like 8-fold motifs (although we can of course never be certain that what we may see as a separate motif was seen in the same way by the original artists), and this is even earlier than Madinat az-Zahra, at 876-79 A.D. There was clearly a long standing tradition of 8-fold, complex motifs preceding the first true rectilinear rosettes. The earliest extant examples of rectilinear 8-fold rosettes date from the first half of the

continued on p. 151 →

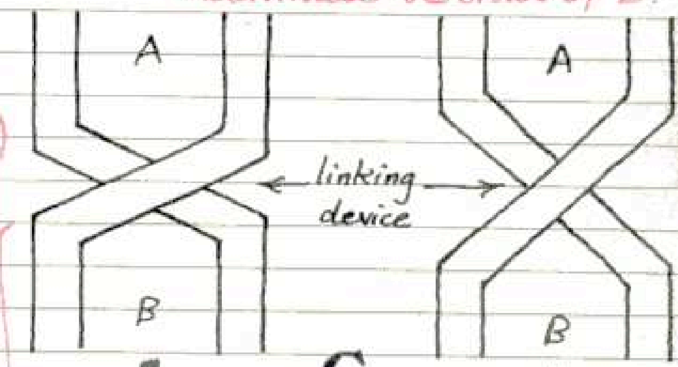


A



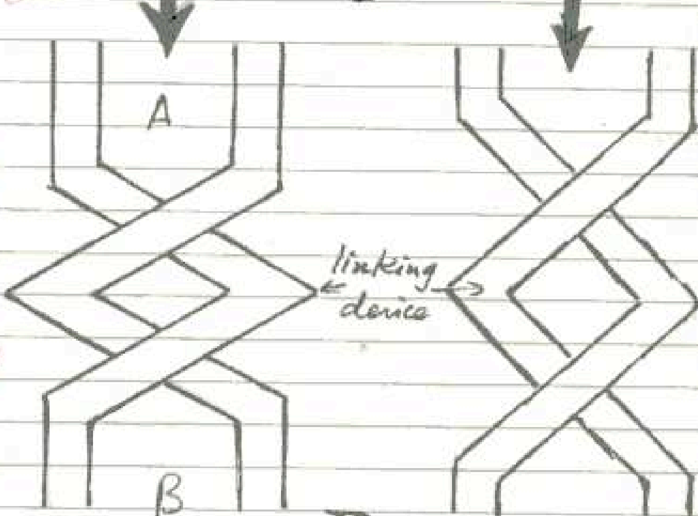
B

Rectilinear versions of B.



C

The linking device here may be termed a <u>crossover</u>. It is the prevalent form of linking device between adjacent motifs or pattern elements in later Islamic rectilinear ornament.
This may become secondarily curvilinear.

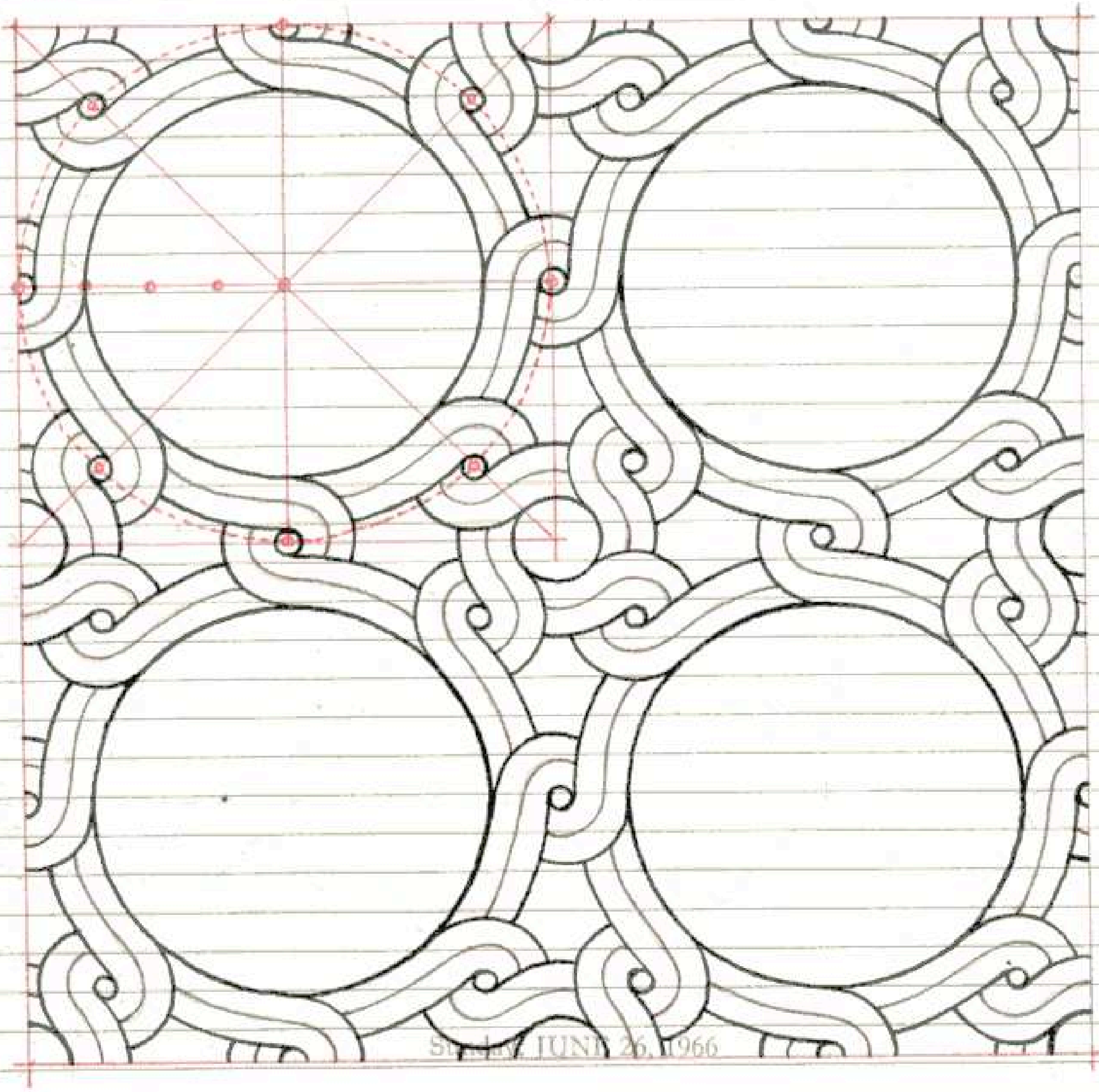


D

Topologically this harks back to the pre-Islamic linking circle of fig. A above.

Tue 22 May 1984

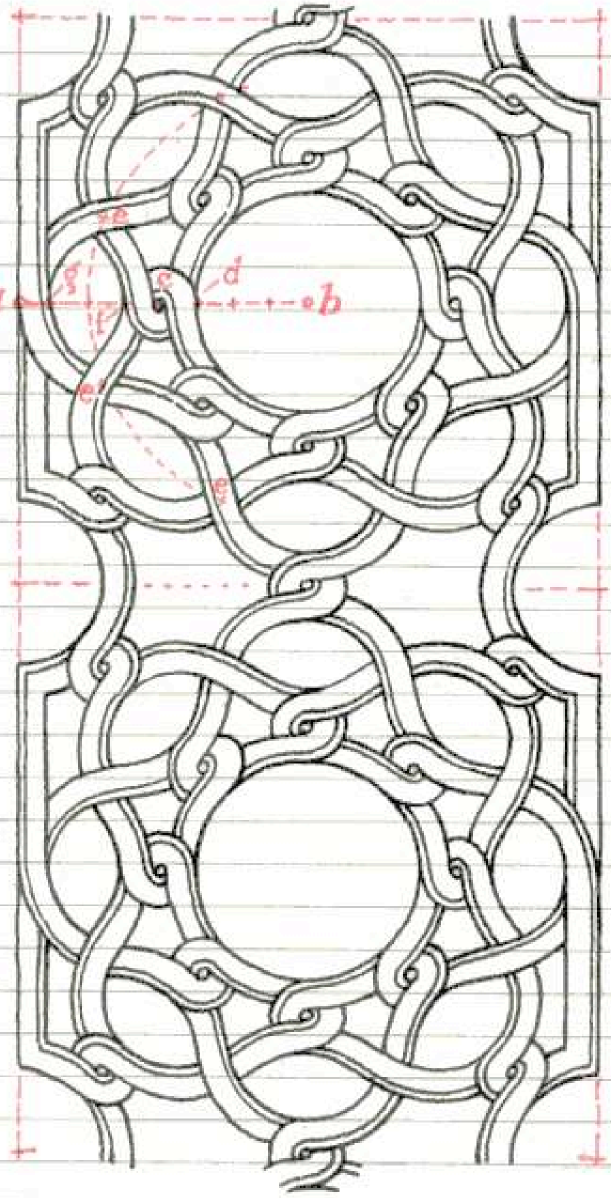
Saturday, JUNE 25, 1966



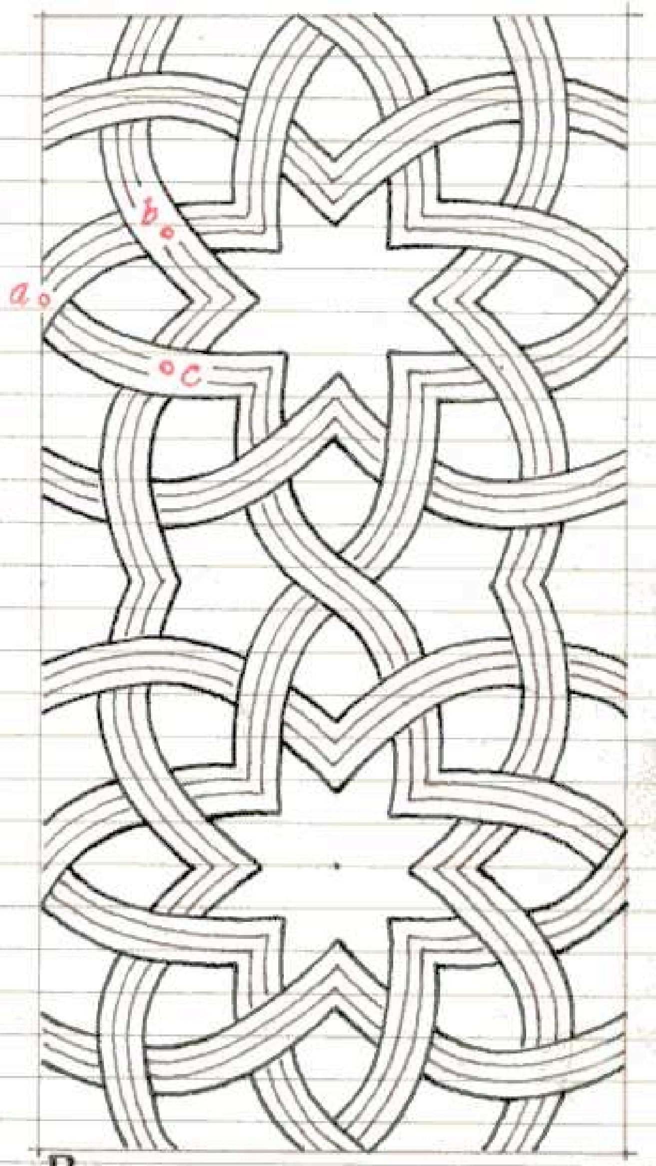
Khirbat al-Mafjar (743 A.D.) Balustrade panel 12A (Hamilton, 1959)

Tue 22 May 1984

Monday, JUNE 27, 1966



A



B

Ibn Tulun mosque, Soffit No 3   876-79 A.D.
Cresswell (1919).

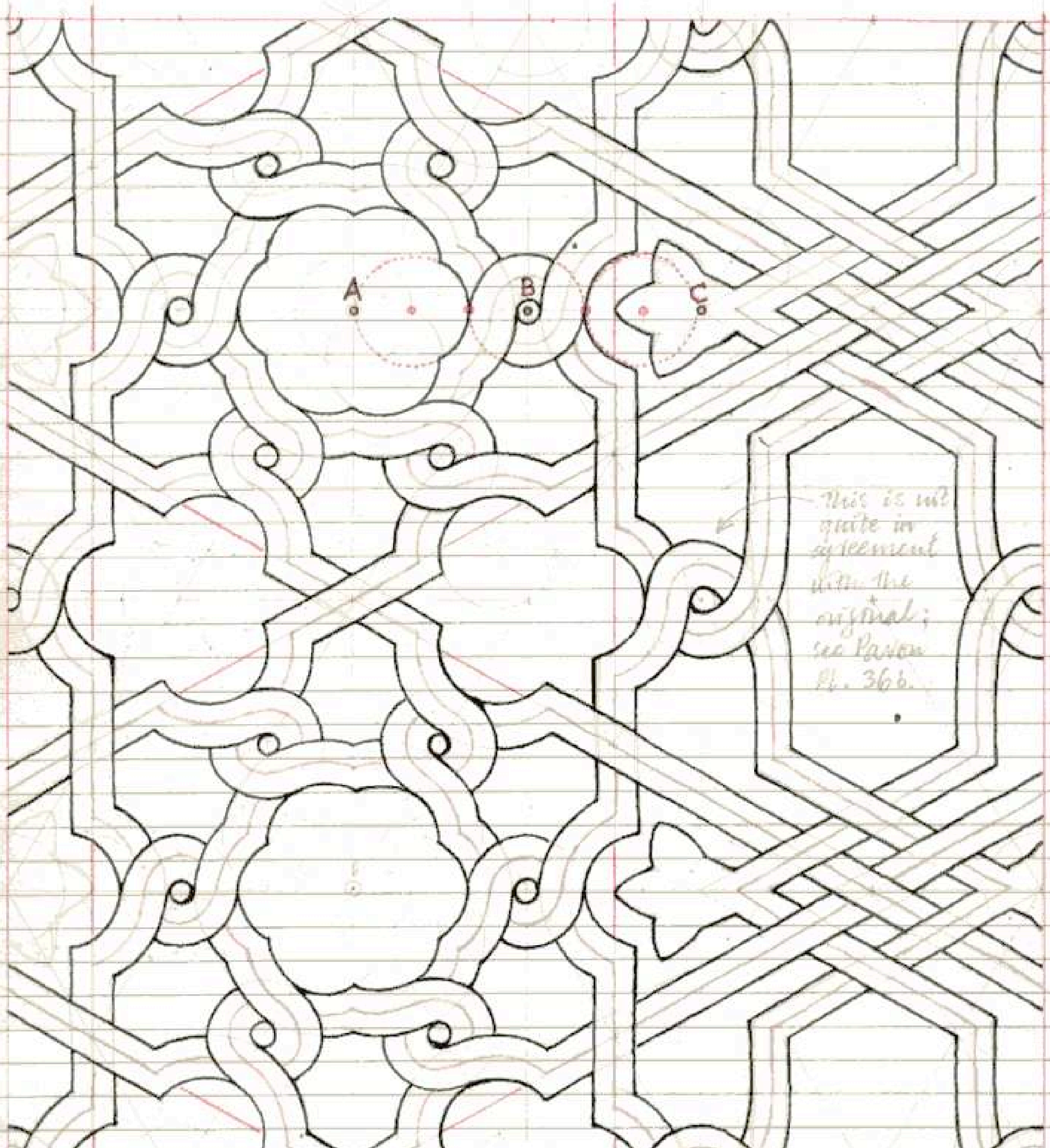
If the circumradius of the 8-fold motif is ab, divide this into 8 equal parts. Then bc = 4 units; bd = 3; be = 6 bf = 5 and bg = 7. The band width is slightly less than 1 unit. These measurements agree with those on Cresswell's original photograph.

Similar to patterns from Khirbat al Mafjar (743 A.D.) and Qasr al-Hair al-Gharbi (727 A.D.).

In the above, points a, b, c are at the vertices of an equilateral triangle. Other proportions differ in authentic sources. Some of these are virtually curvilinear versions of the star & cross pattern. A straight line topological equivalent of the above exists in one window of Qasr al-Hair al-Gharbi.

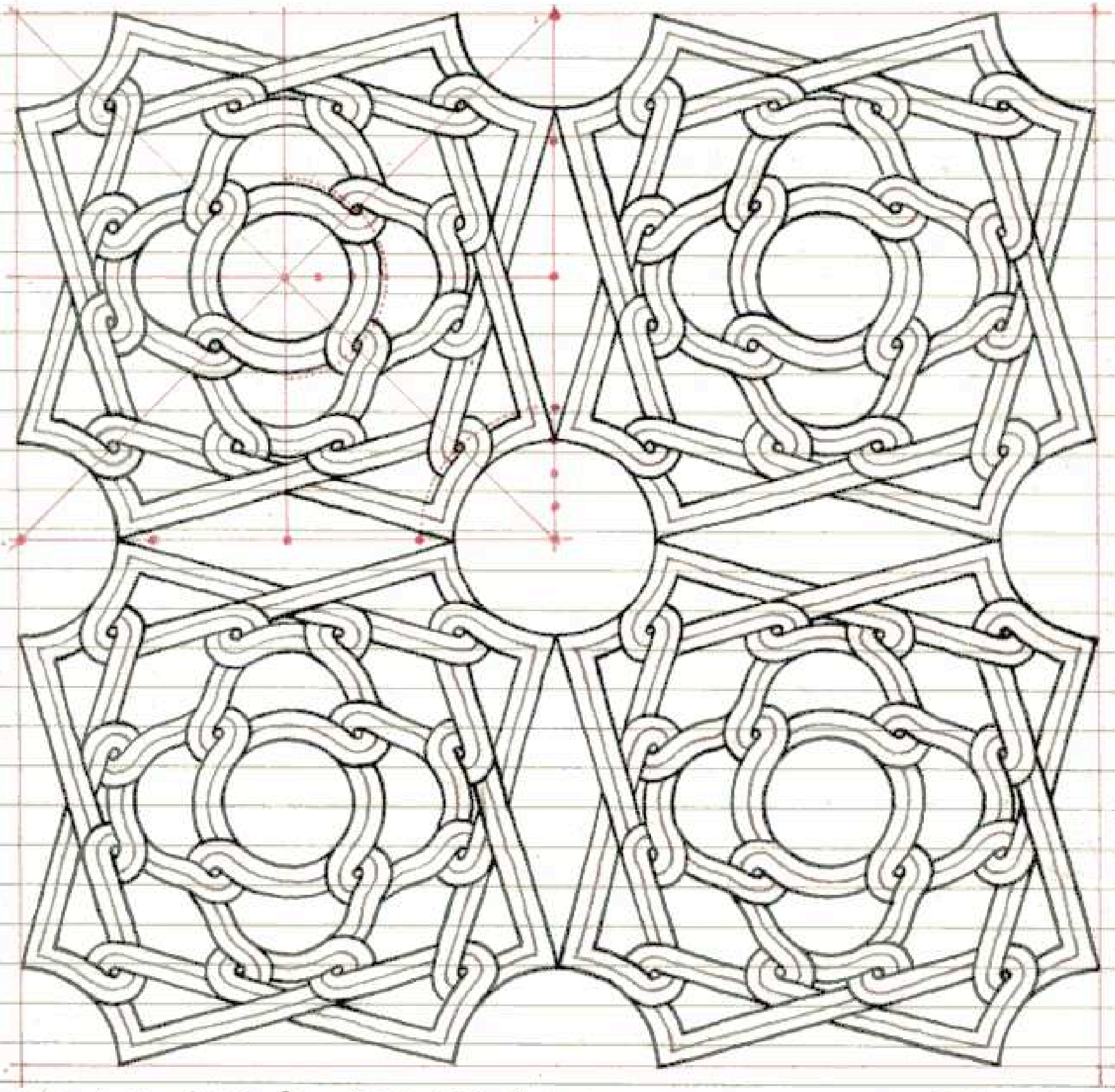
Wed 23 May 1984

Tuesday, JUNE 28, 1966



Madinat al-Zahrā' (936–76 A.D.) Original construction, based on measurements of photographs in B. Pavon Maldonado (1975) "El Arte Hispano-Musulman...." Pls 31, 36. His own drawing (p. 96, fig. 33) is an inaccurate and badly drawn sketch.

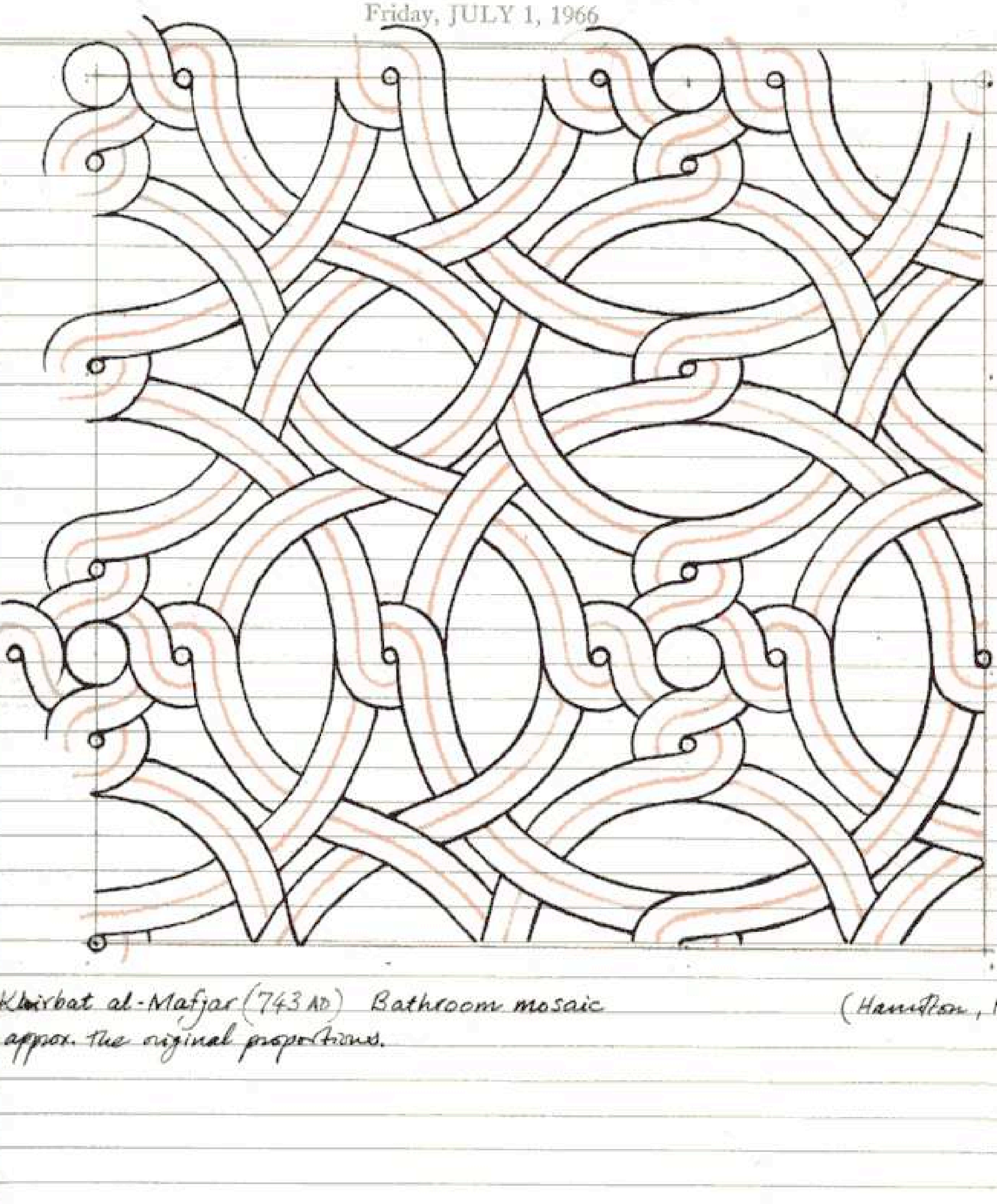
Tue 22 May 1984



Khirbat al-Mafjar (743 AD) Bathroom mosaic No 25 (Hamilton, 1959)

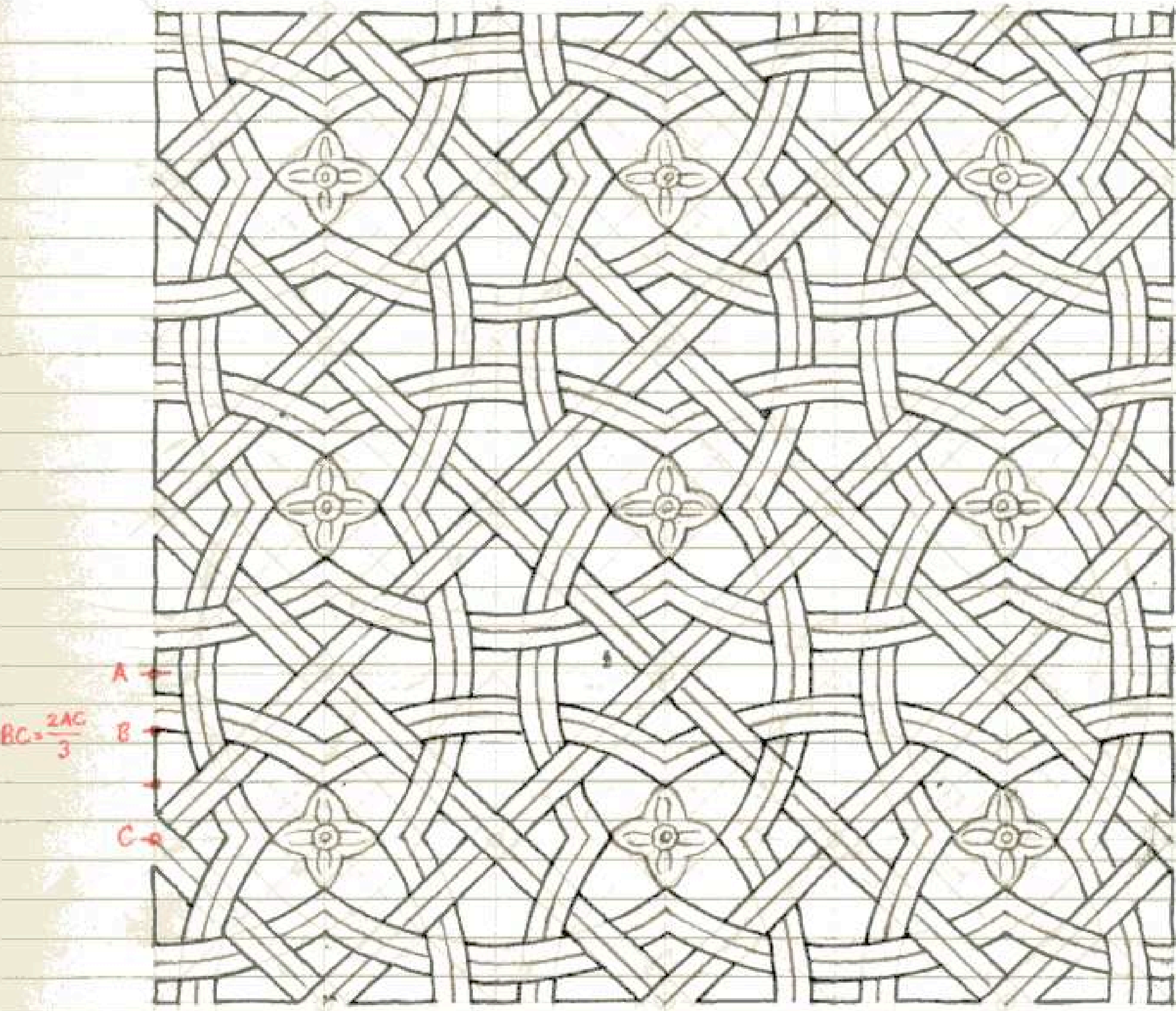
RJ Lee Wed 23 May 1984

12th century, although 10-fold rosettes occur in the North Dome chamber of the masjid-i jami at Isfahan (1088 A.D.), and one would have expected 8-fold rosettes to have appeared before 10-fold, since patterns of 8-fold rosette-like motifs were common centuries before the end of the 11th century. All we can say at present therefore is that the first true geometrical rosettes, probably 8-fold, may have appeared some time between the date of the Arab-Ata mausoleum (977 AD) and the end of the 11th century. It is just possible that a thorough search through the relevant literature might reveal examples of the occurrence of rosettes well before the end of the 11th century, but on the other hand the effort required may not be worth the candle. For the purposes of my own studies in any case, this is a pure marginal question, and any discussion of historical origins would only be touched on for the sake of general interest. There is enough material for a purely geometrical study of these star patterns to last a whole lifetime of research, without any involvement in any way in their place in Islamic history and culture.



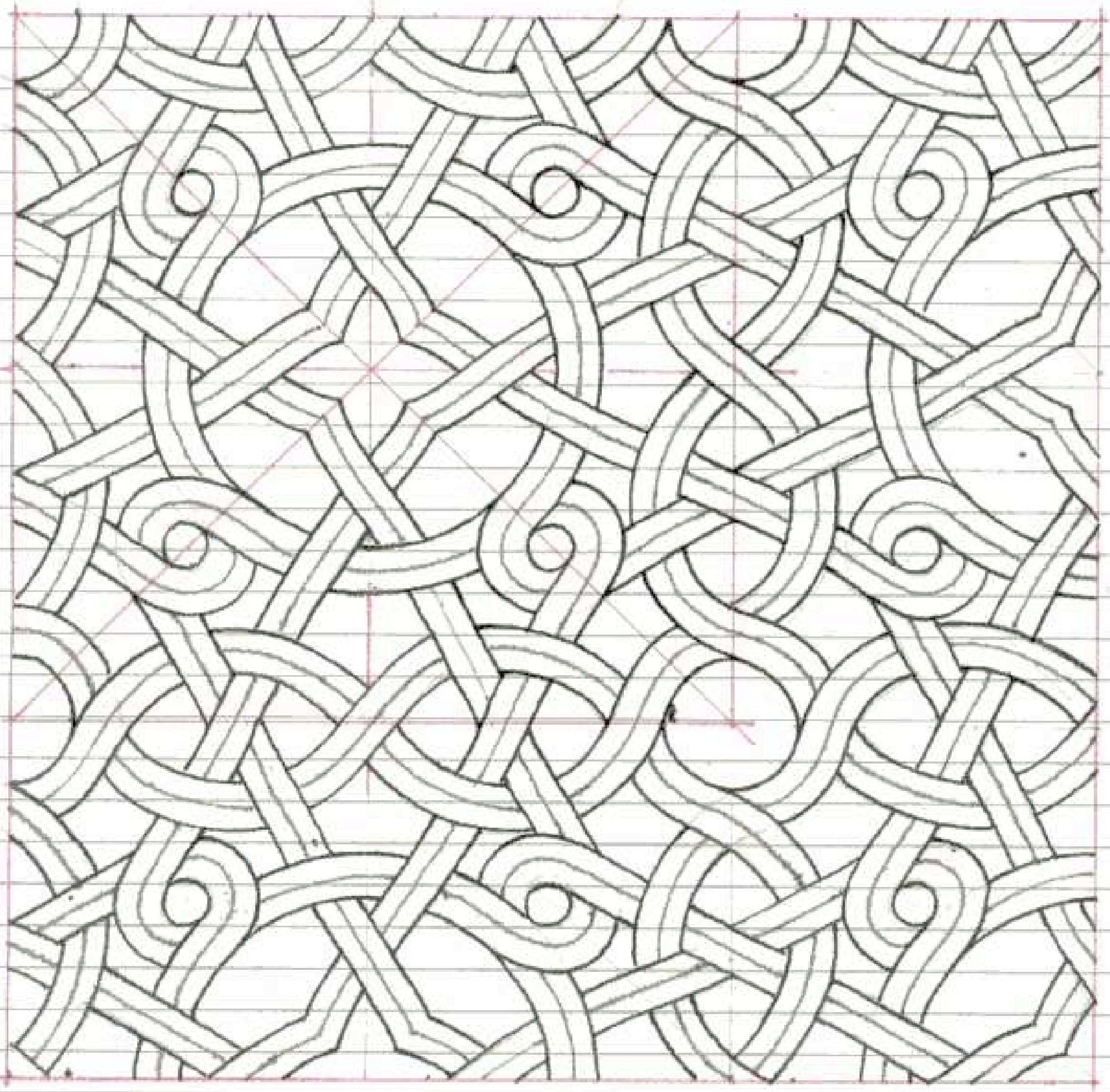
Khirbat al-Mafjar (743 AD) Bathroom mosaic (Hamilton, 1959)
approx. the original proportions.

A Hzu Thu 24 May 1984



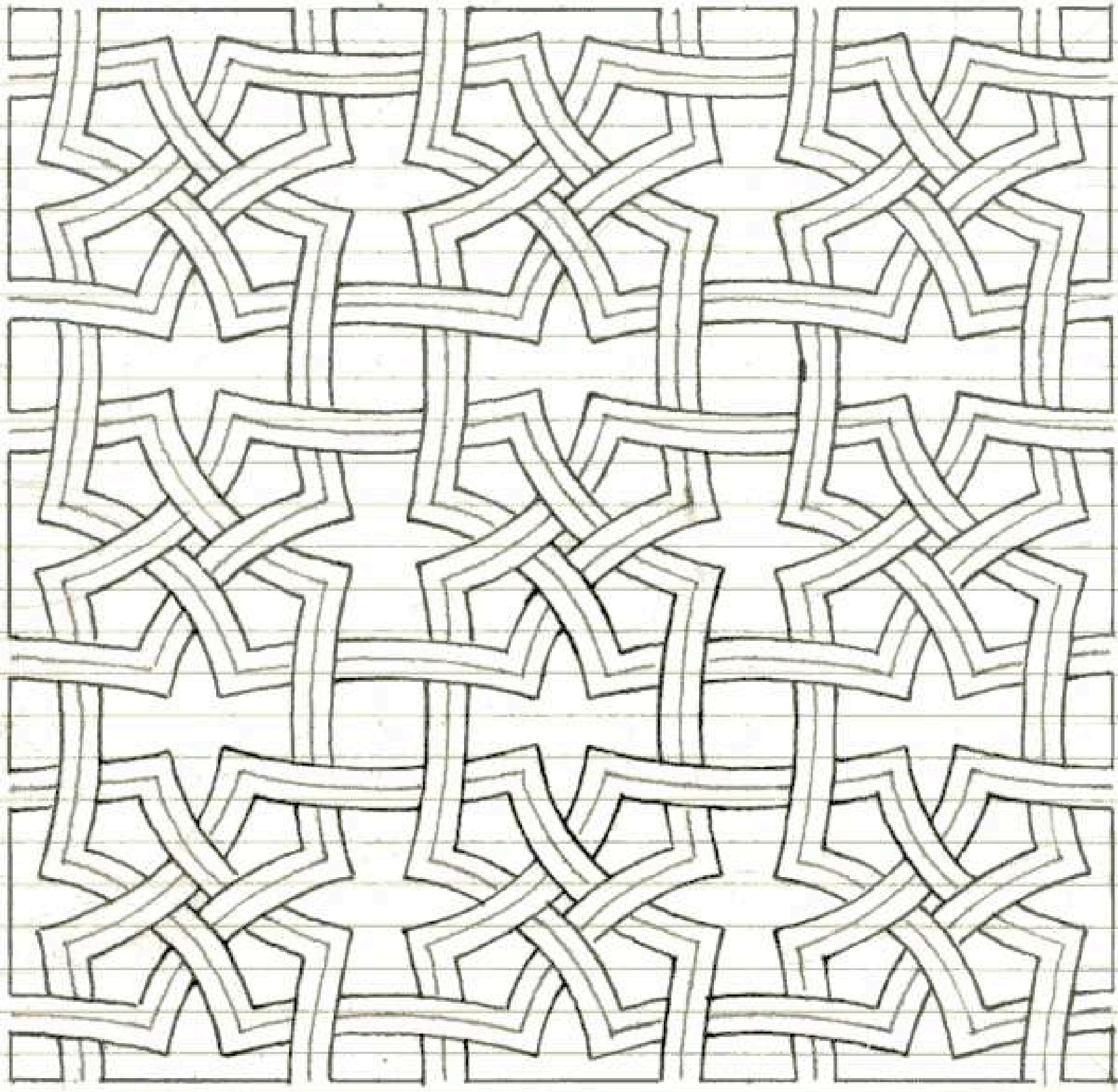
Qaṣr al-Ḥair al-Gharbi (727 A.D.) Window grille (K. Brisch 1966, Pl. 60b) (S. Abdul-Hak, 1951, Plate 17 top).

Thur 24 May 1984



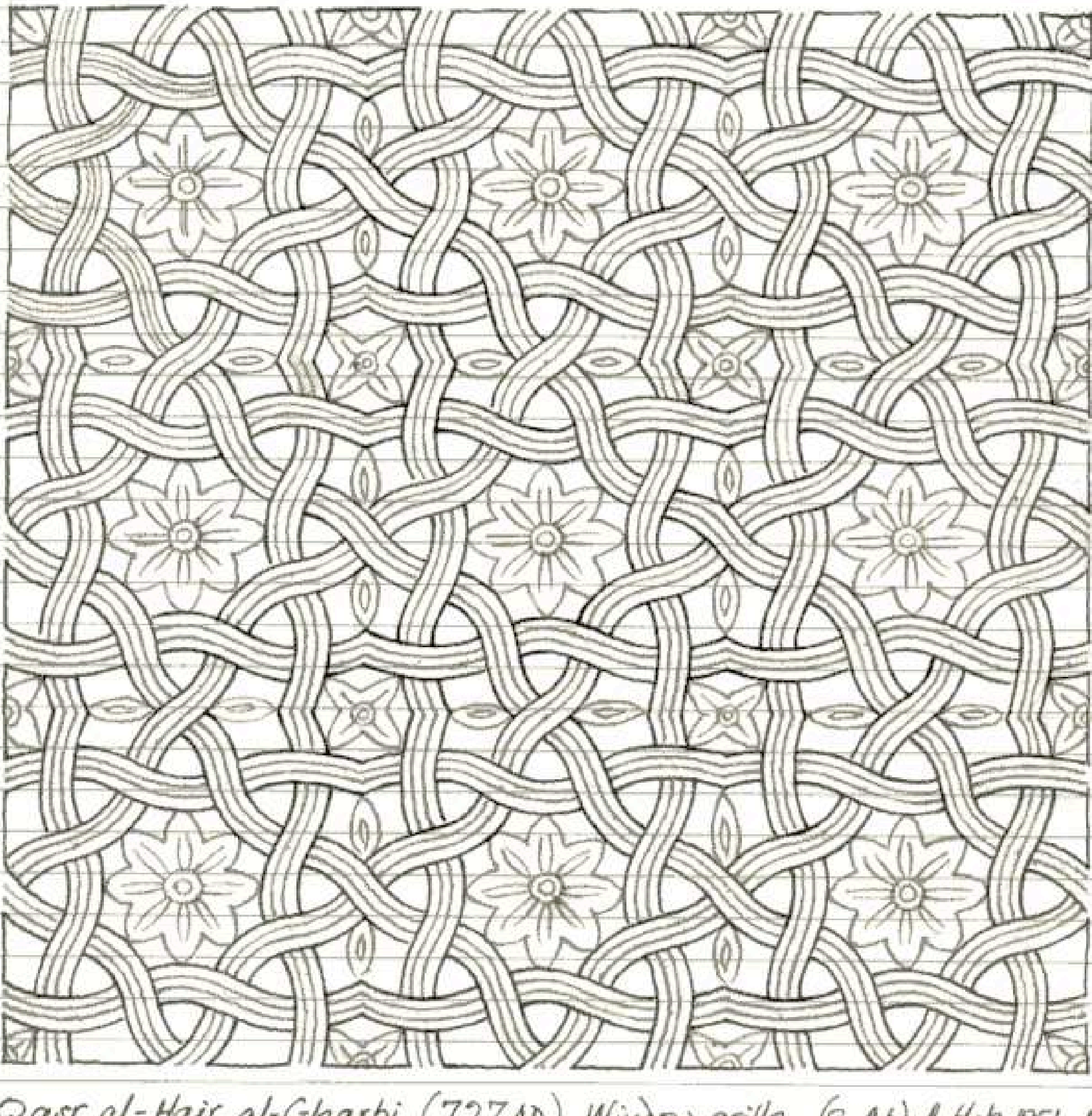
Qaṣr al-Ḥair al-gharbi (727 AD) window grille (K. Brisch 1966, Pl. 71b)

A.Aez Thu 24 May 1984



Qasr al-Hair al-Gharbi (727 AD) window grille (S. Abdul-Hak 1951, Pl. 23 bottom)

The above drawing does not exactly correspond to the original, which is obviously a curvilinear version of the inverse star-and-cross pattern. The curvilinear octagons should perhaps have been regularly drawn.



Qasr al-Hair al-Gharbi (727 AD) Window grille (S. Abdul-Hak 1951, Pl. 23 top) (see also K. Brisch 1966 Pl. 72b).

A similar pattern is found at Khirbat al-Mafjar (743 AD) (Hamilton 1959, p. 289 fig. 246 window P5) but the inner angles of the 8-fold "rosettes" are produced as cusps, and instead of curvilinear octagons there are curvilinear 4-stars.

LINKS BETWEEN ADJACENT STAR-CENTRES A Thu 24 May 1984

See p. 24 for definition of "star-centre".

Thursday, JULY 7, 1966

If the outer points touch, the link is direct; if not, indirect

1. Collinear Links — fig. 158 A, B — principal or secondary radii of two adjacent star-centres are collinear
   a) Primary, or radial collinear link: The centres a and the outer points b of the two stars lie on a single straight line. The stars may or may not share a single outer point.

   b) Secondary, or interradial collinear link: an interradius of each star lies on the same straight line.

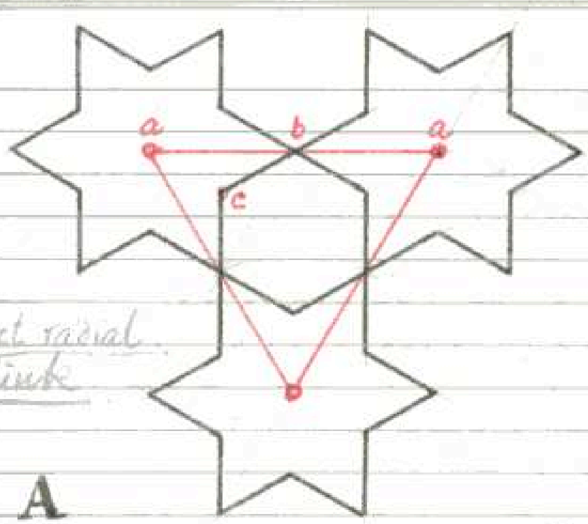
   c) Mixed collinear link (not illustrated) see pp. 161, 162, 164: two adjacent star motifs may form a collinear link involving the principal radius of one and a secondary, or interradius of the other.

2. Parallel Links — fig. 158C - The nearest radii to the line joining the centres of two adjacent stars are parallel to one another, but do not coincide with that line. The angle through which the stars are rotated with reference to the line joining their centres is usually "fixed" in some way by the geometry of the pattern and can usually be precisely determined by trigonometrical or other means. See pp. 193, 194 for the use of parallel links in rhombic arrangements of 10-stars.

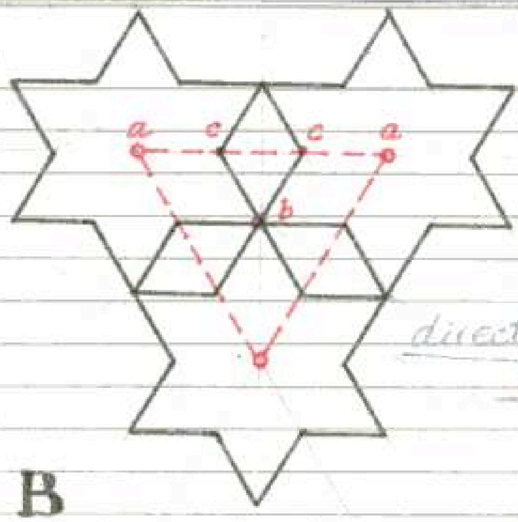
3. Skew Links — fig. 158D — neither principal radius nor interradius coincides with or is parallel to the line joining the centres of two adjacent star-centres. It is possible for two stars forming a skew link to share an outer point b, as shown in figure D. This latter configuration however, is not suited to interlace, particular with rectilinear stars, since it results in a sudden discontinuity of direction at the point of the crossover b. Ideally a rectilinear crossover should present 2-fold rotational symmetry locally, about the crossing over point. This is in fact so in most authentic rectilinear ornament but examples of discontinuous crossovers are not uncommon.

NOTE: In some of my earlier notes collinear links are called "alignments", whereas parallel links are called "dislocations".

direct radial link



direct interradial link

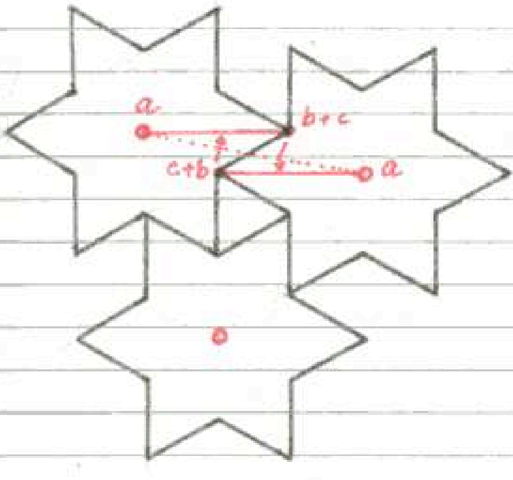


A

Collinear Links – primary radii
or Radial collinear link.

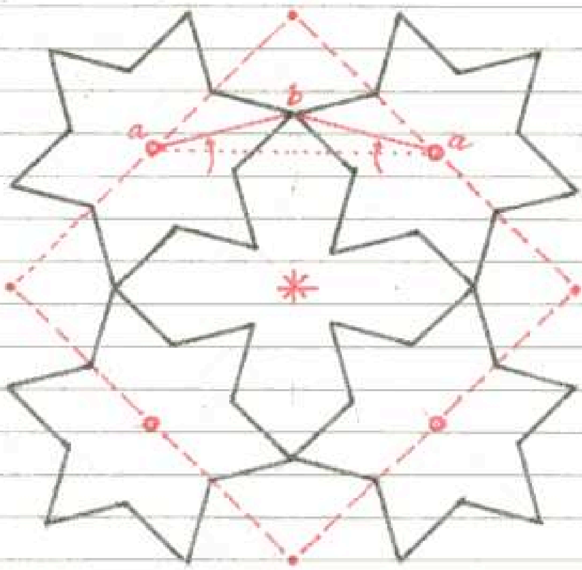
B

Collinear Links – secondary radii
or Interradial collinear link.



C

Parallel links – centres form an equilateral triangle



D

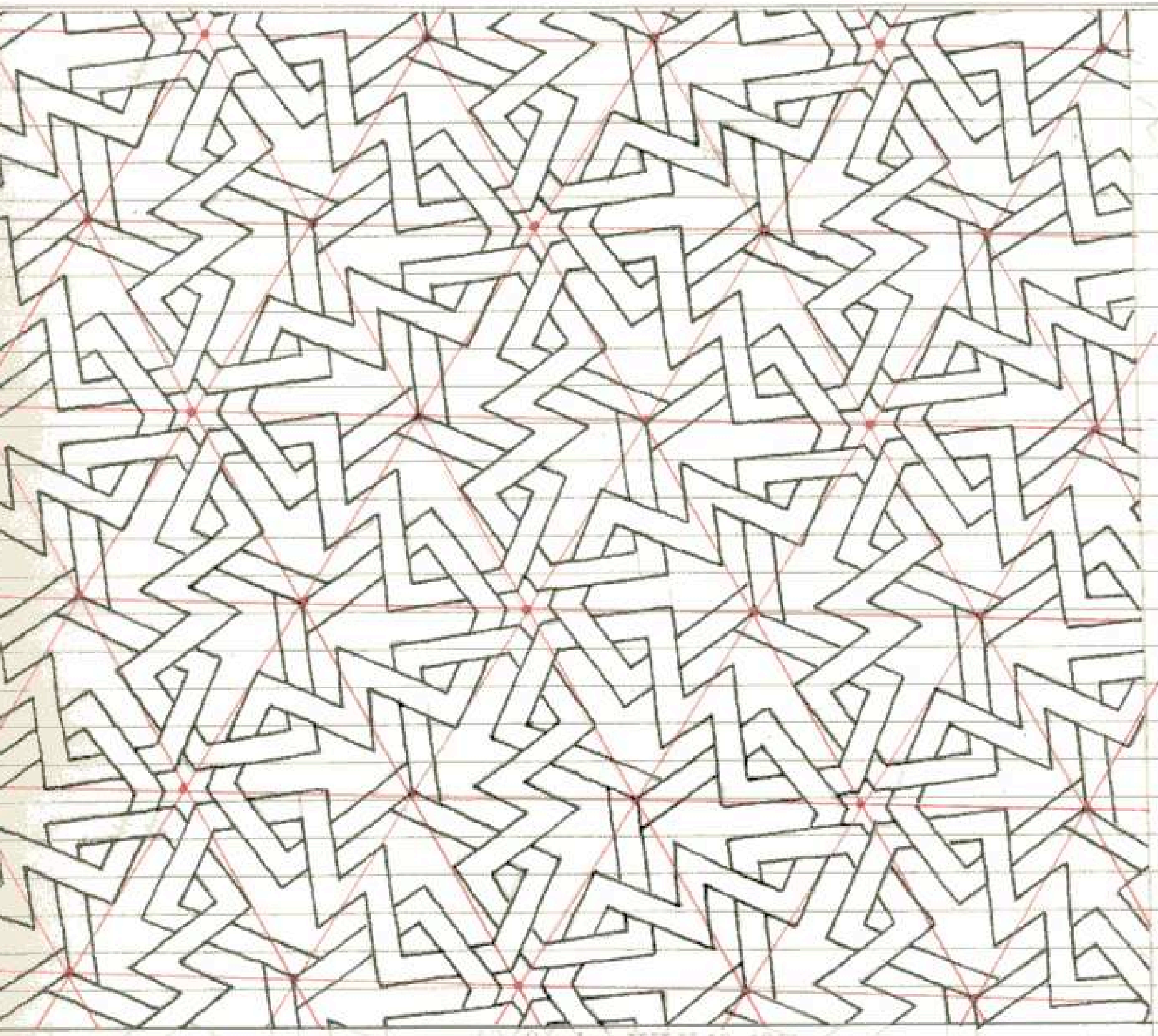
Skew links – centres form a square.

The four major classes of links between adjacent stars – all the above arrangements occur in authentic patterns using 6-stars. In parallel & skew links both stars are rotated by the same angle with respect to the line joining their centres. In parallel links the sense of rotation is the same in both stars. In a skew link the stars are rotated in opposite senses, through the same angle.

59 | Parallel Link : 6-stars

Thu 24 May 1984

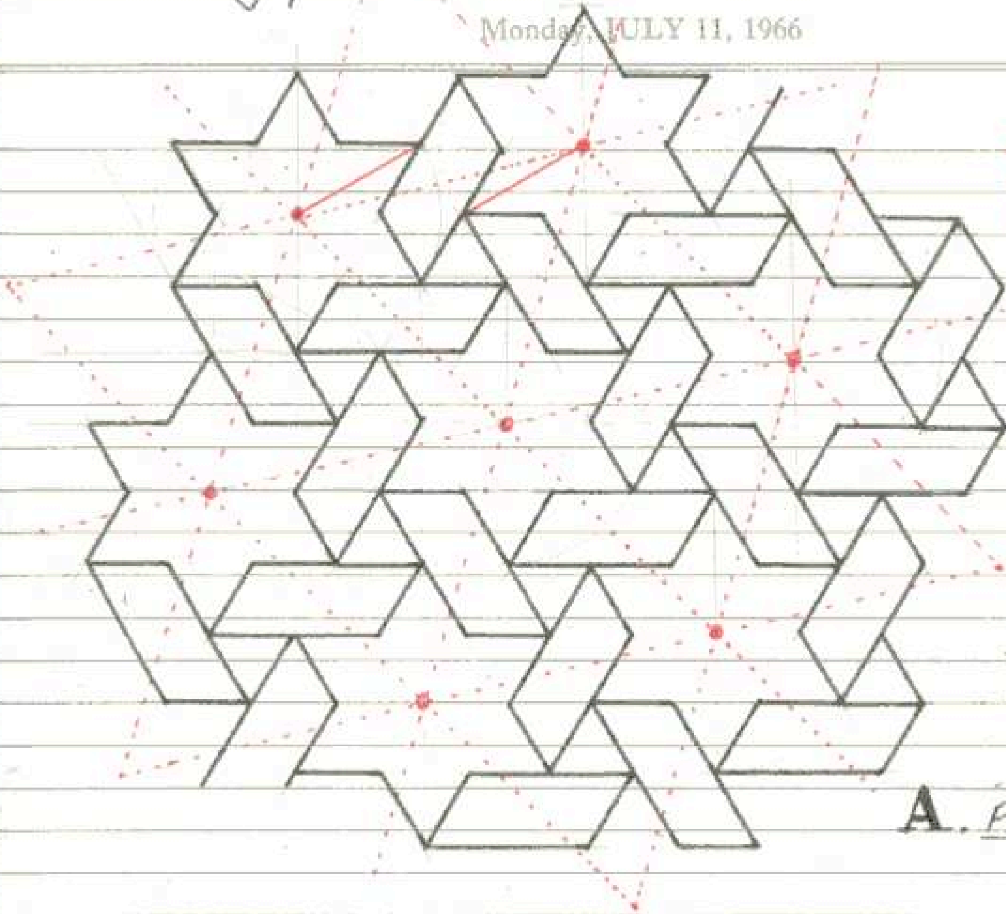
Saturday, JULY 9, 1966



Sunday, JULY 10, 1966

Pattern on brick mihrab of later Kharragan tomb tower (1093) involving parallel links between 6-stars. Since 3-way nodes are present the pattern cannot be a true interlacing pattern.

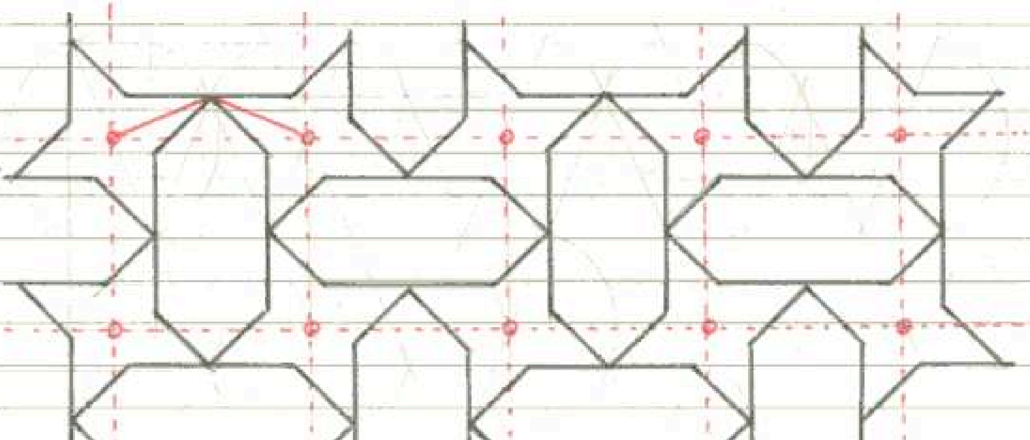
A satisfactory copy of this pattern can be constructed entirely on a grid of small triangles, as was the above drawing.



In the original pattern the 6-stars are outlined by 2x1 bricks shaped like parallelograms

Occurs on the lates of the two Kharraqan tomb towers

A. Parallel Links - 6-sta

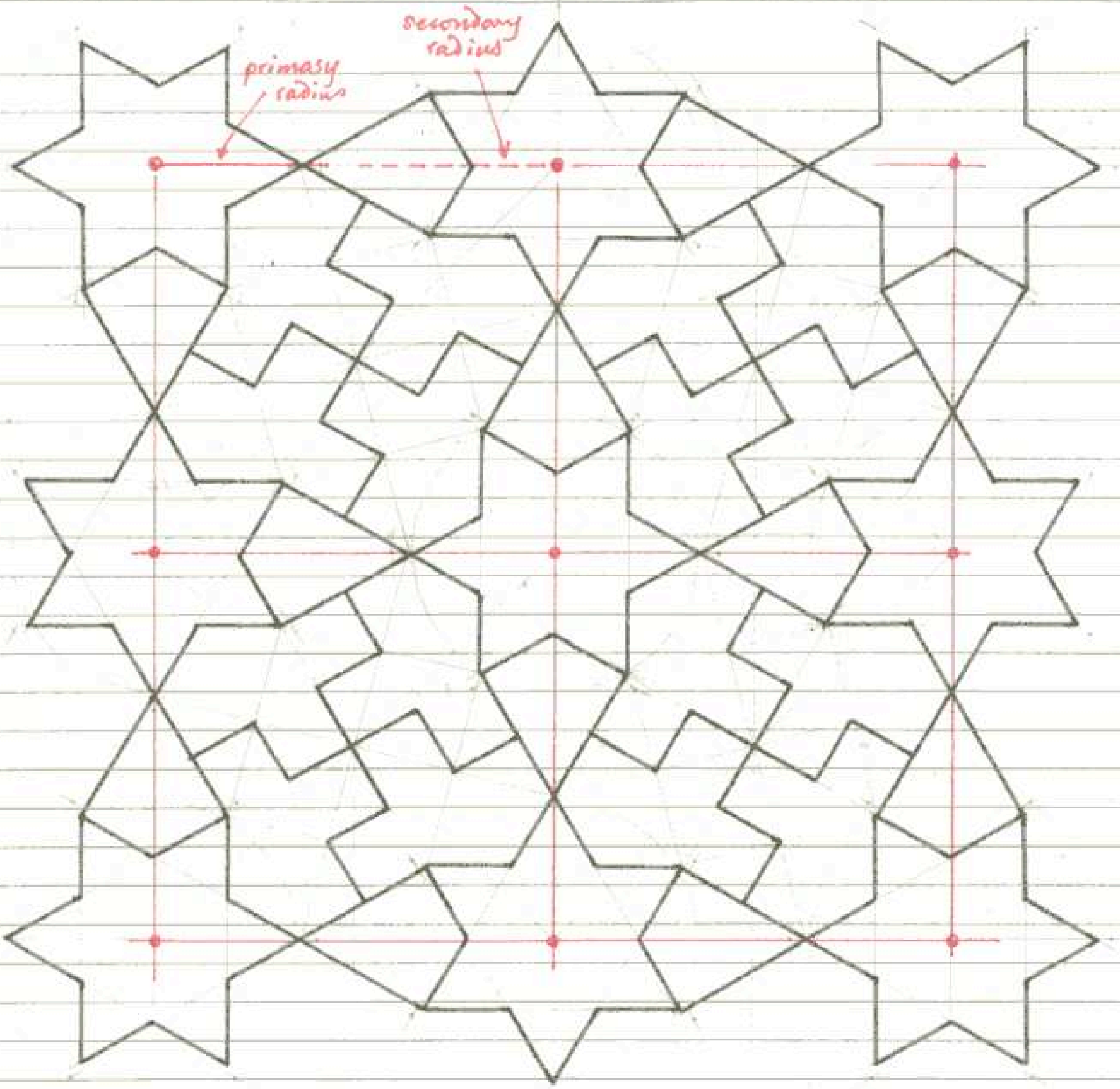


B. Skew Links between 4-stars.

This occurs on the lates of the two Kharraqan tomb towers.
The 4-stars on the original are outlined by bricks shaped like elongate hexagons.

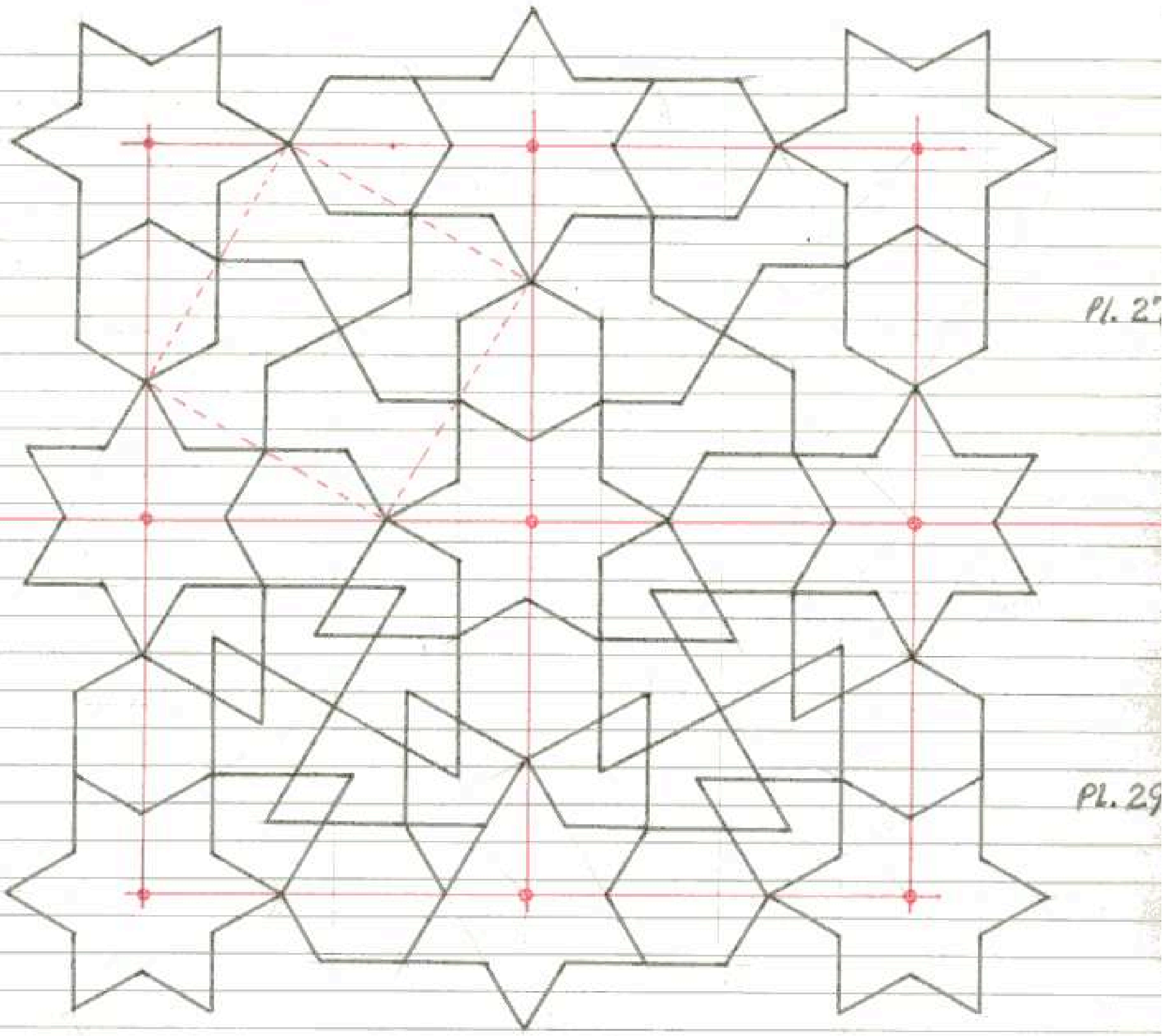
"MIXED" COLLINEAR LINKS

AJLee Fri 25 May 1984



Mixed Collinear Links between 6-stars (original, but very likely exists as an authentic pattern; Bourgoin's (1879) Plates 27–30 are on the same basis, Plates 27 and 29 contain 6-stars and are superposable on the pattern shown above – see p. 162 opposite).

Wednesday, JULY 13, 1966



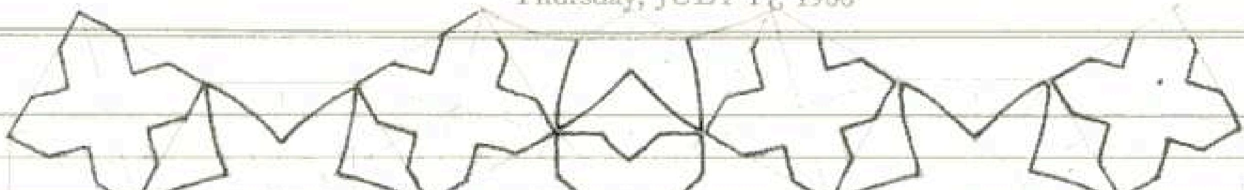
Pl. 2

Pl. 29

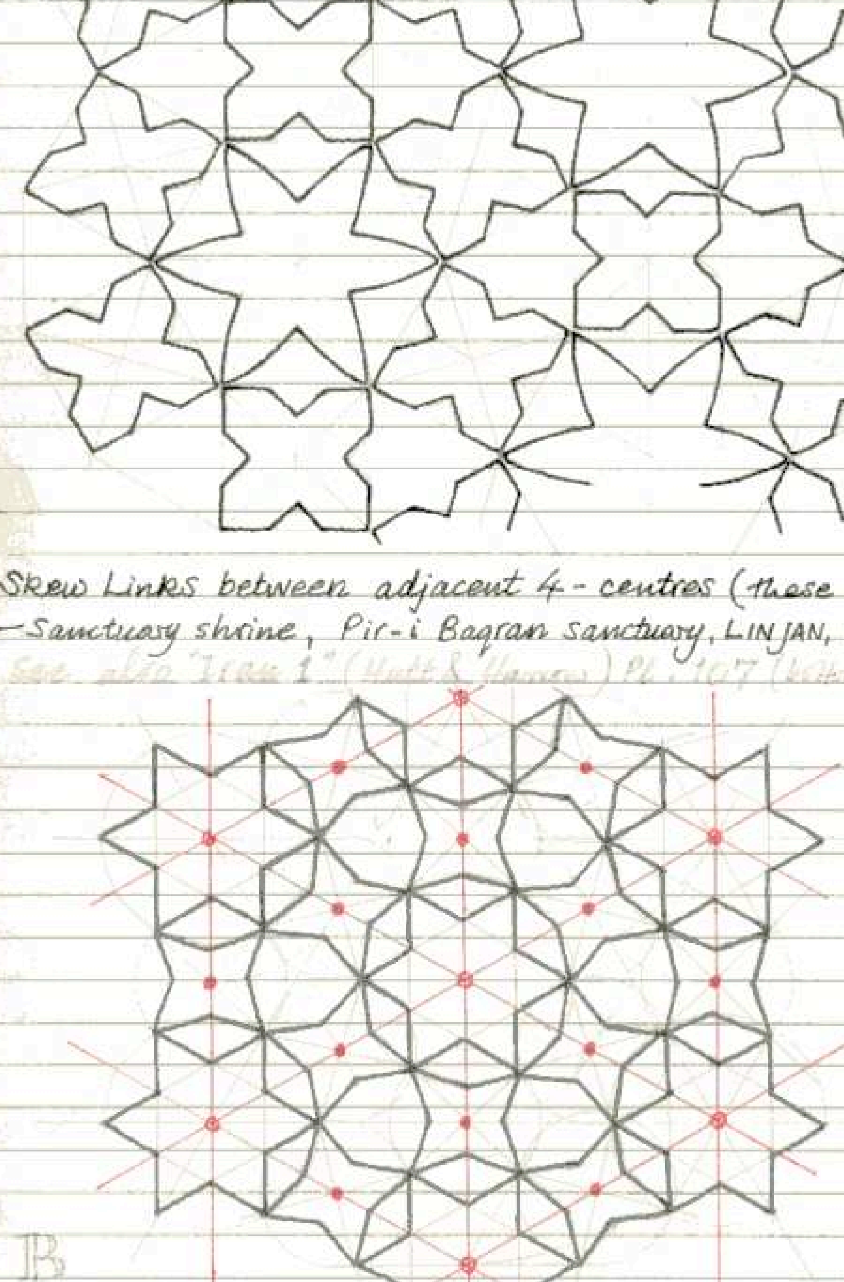
<u>Mixed Collinear Links</u> between 6-stars – from Bourgoin (1879)

All three of these patterns, together with Bourgoin's Pl. 28, are related to an underlying snub tessellation $s\{{4 \atop 4}\}$, one square of which is indicated by broken lines at top left, above.

Fri 25 May 1984



Skew Links between adjacent 4-centres (these are strictly 4-rosettes, cf. p. 108) — Sanctuary shrine, Pir-i Bagran sanctuary, LINJAN, Nr. Isfahan (H&G fig. 288).

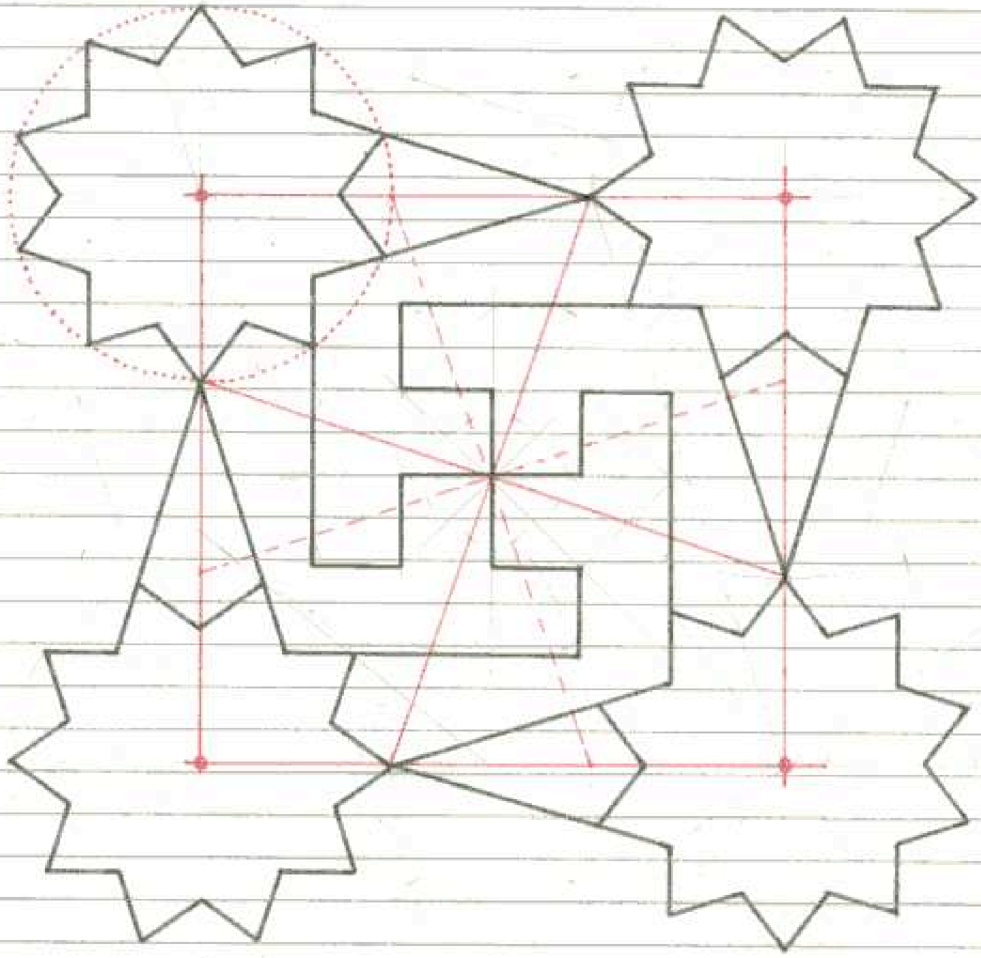


Skew links between adjacent 4-stars. These are centred on mid-edges of {3, 6}.

(original, but obviously the basis is identical to fig. A above).

Wed 30 May '84.

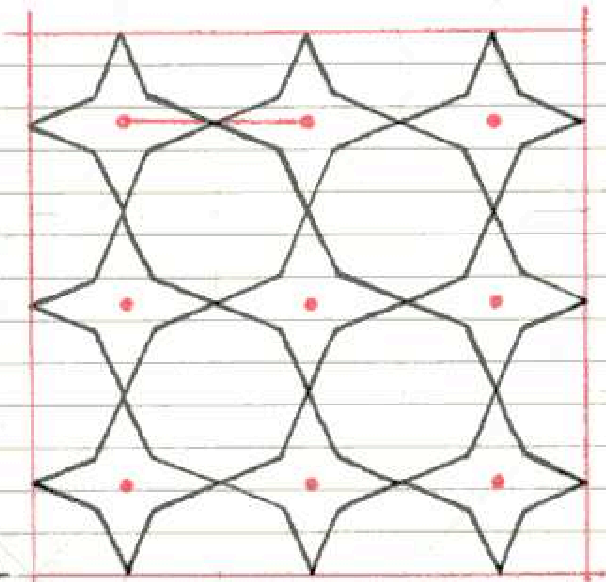
Friday, JULY 15, 1966



Mixed Collinear Links between 10-stars: Isfahan, prayer eyvan in Friday mosque (14-15 cent) - see H. & G. figs. 310, 311.

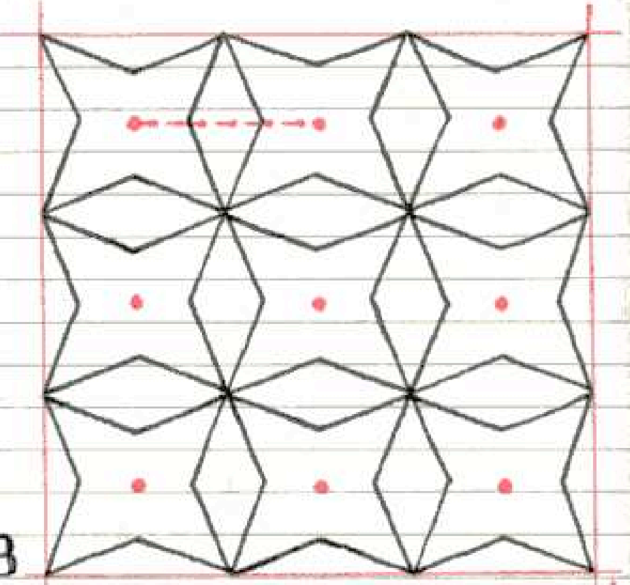
Tue 29 May 1984

Saturday, JULY 16, 1966



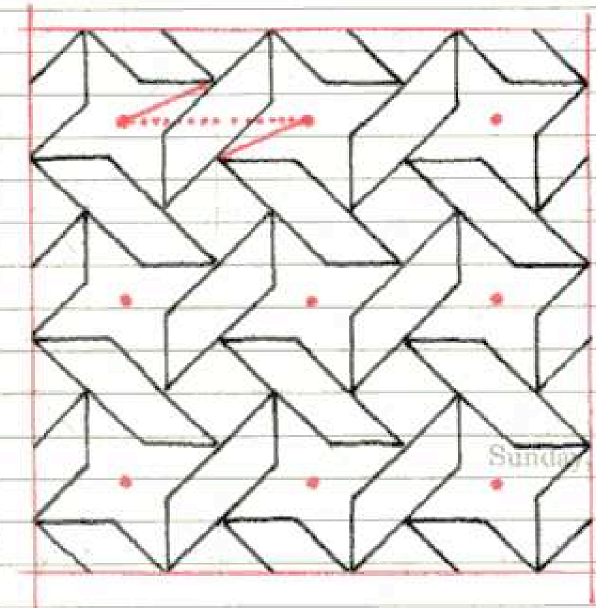
A

Radial collinear links widespread



B

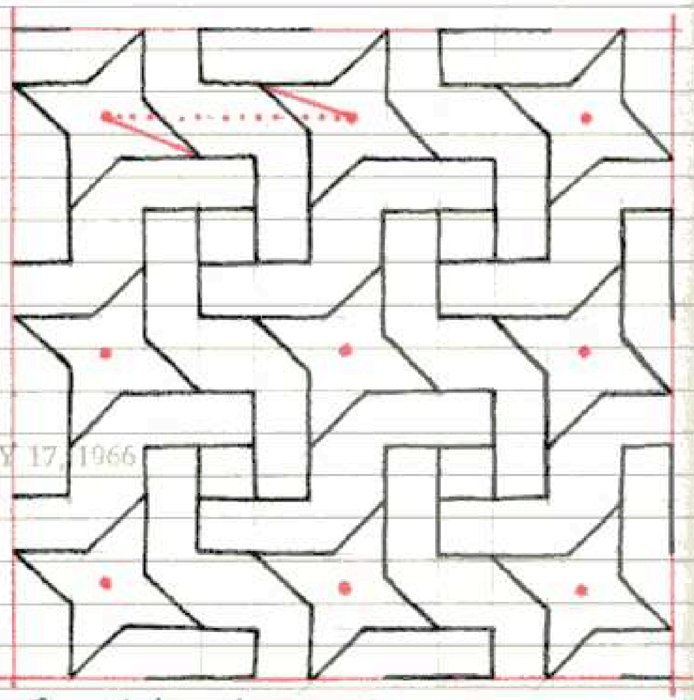
Interradial collinear links widespread



C

Parallel links Centr. Asia

Sunday, JULY 17, 1966



D

Parallel links Asia Minor

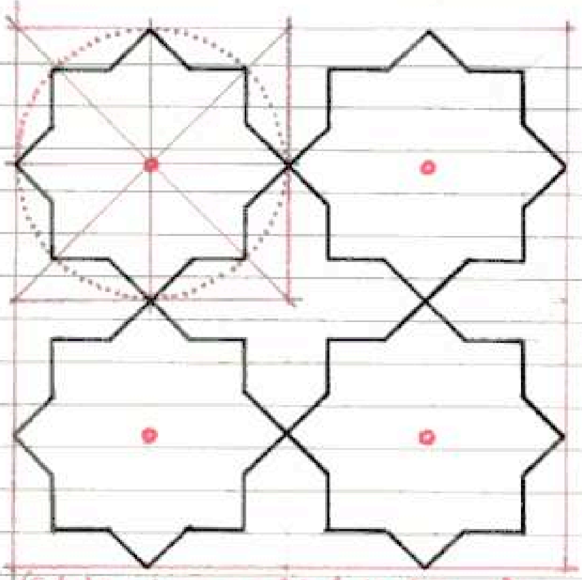
fig. 165

Examples of Collinear and Parallel Links with 4-stars

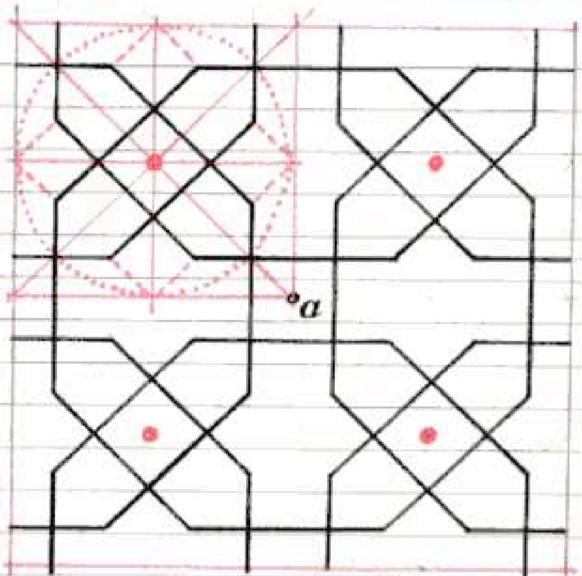
N.B. Owing to the placing of these 4 stars on tetrads, radial collinear links are necessarily present simultaneously with interradial links

Tue 29 May 1984

EARLY EXAMPLES OF PATTERN DERIVATION



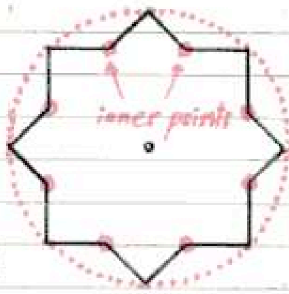
A



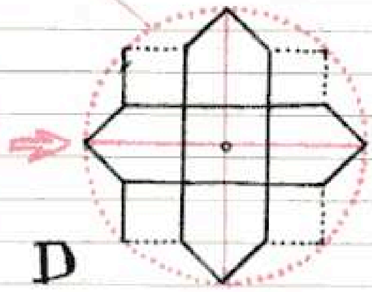
B

(8/2)1 stars – radial links

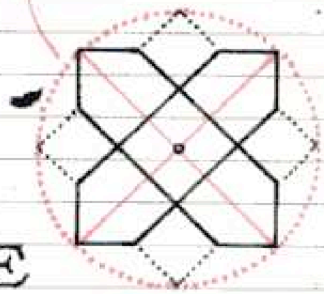
4-"rosettes" – interradial linked and indirect radial links



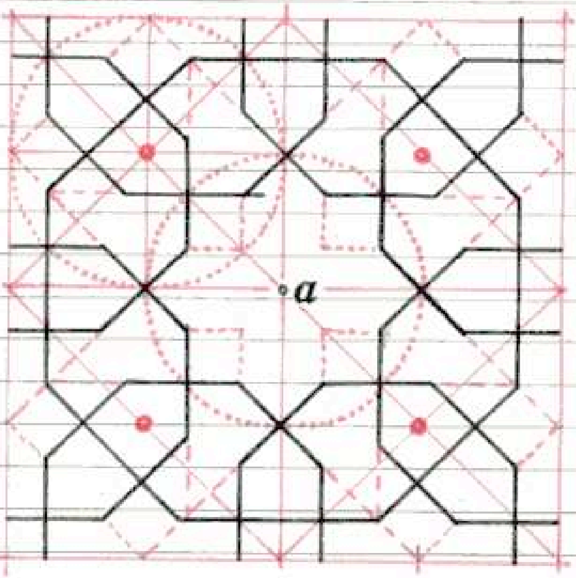
C



D



E



F

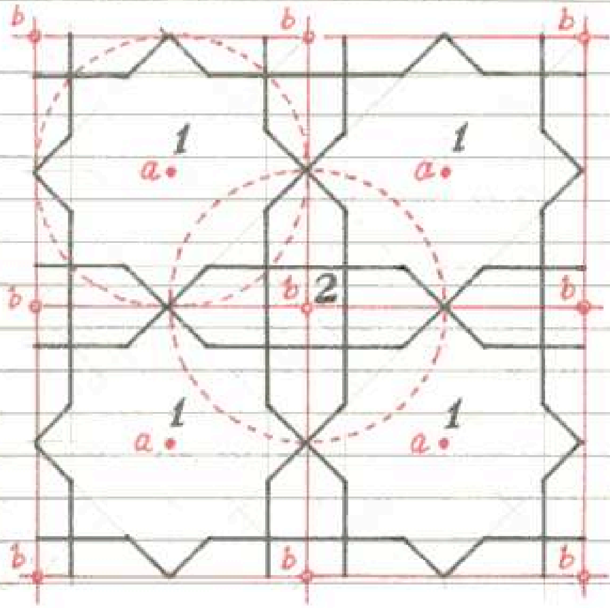
A is the widespread "star & cross" pattern, one of the oldest star patterns in Islamic ornament, adopted from classical sources. By turning the 'free' points inwards on each (8/2)1 star two configurations are possible (D, E). D leads to another star & cross pattern, while E gives rise to the inverse star & cross pattern at B. This consists entirely of overlapping, interlaced regular octagons.

From B pattern F is easily derived, by placing an identical (8/2)1 star at point a, and eliminating certain pattern lines. This pattern appeared early in the Middle East and Central Asia, probably no later than about 1100 AD*, along with many similar variations. * or early 11th century.

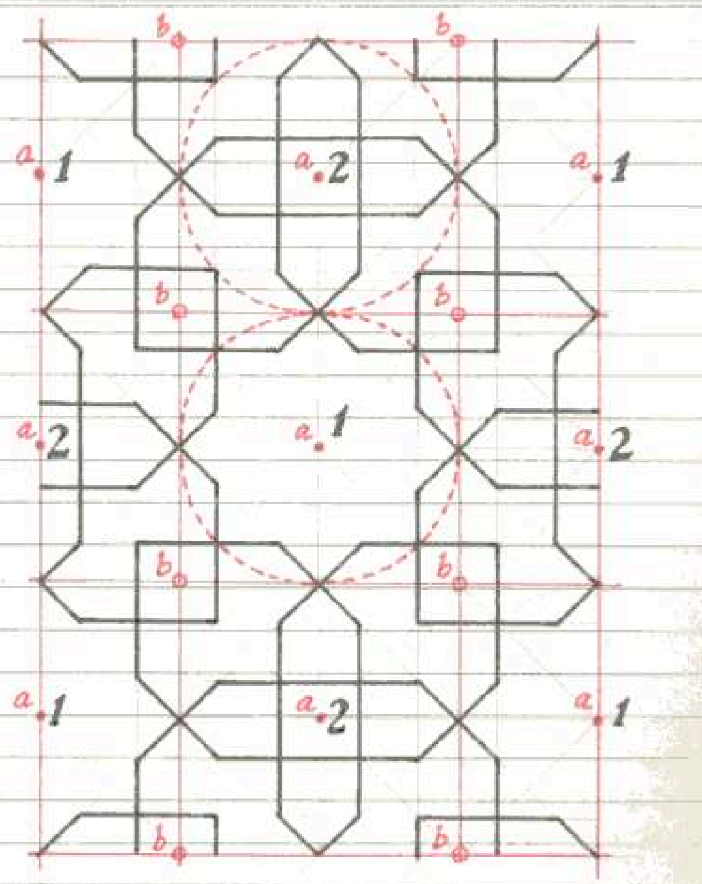
Wed 30 May 1984

The earliest surviving example of the "Star & cross" pattern (fig. 168 A) in Islamic ornament seems to be as a window grille from the palace of Qasr al-Hair al-Gharbi in Syria (727 A.D.). It also occurs in 9th century Samarra, in the 10th century palace of Madinat az-Zahra near Cordoba, and in Al-Hakim's mosque in Cairo (1003 A.D.). The pattern consists of (8/2)1-stars (1) centred on points a, and 4-pointed crosses (2) centred on points b. The circumradii of both motifs are identical.
The first example in which any attempt has been made to experiment with this simple pattern, in order to derive new patterns from it, is a brick and stucco pattern (fig. 168 B) from the tomb of Nasir bin Ali (1012-13 A.D.). Here, the stars (1) on alternate points a are replaced by crosses (2), the two forming direct radial links. Centres b now come to be enclosed by small squares identical to those at the centres of the crosses. From the beginning of the 11th century this kind of experimentation must have continued at an increasing pace with many kinds of simple ornament. One of the earliest and more successful examples of this trial and error experimentation is shown in fig. 168 C. This survives on a number of early minarets in Iran and Afghanistan dating from about 1050 to just after 1100 A.D. in brick ornamentation. Also from the early 12th century is a further modification of this same pattern (fig. 168 D) from the Maghak-i Attari mosque in Bukhara.
As mentioned on p. 166 the pattern of fig. 168C can be more directly derived from the inverse star & cross pattern (fig. 166 B) rather than from the star & cross itself. A purely incidental feature of the inverse pattern is that it consists entirely of overlapping regular octagons, and since these are fairly small relative to the repeat distance of the pattern this feature must have quickly impressed the earliest artists who drew this pattern. It is the observation of this feature which leads directly to the pattern of fig. 168C, the emphasis being on one octagon with a new 8-star at its centre, on point b.
The derivation of this pattern from the inverse star & cross presupposes the prior existence of the latter. A purely

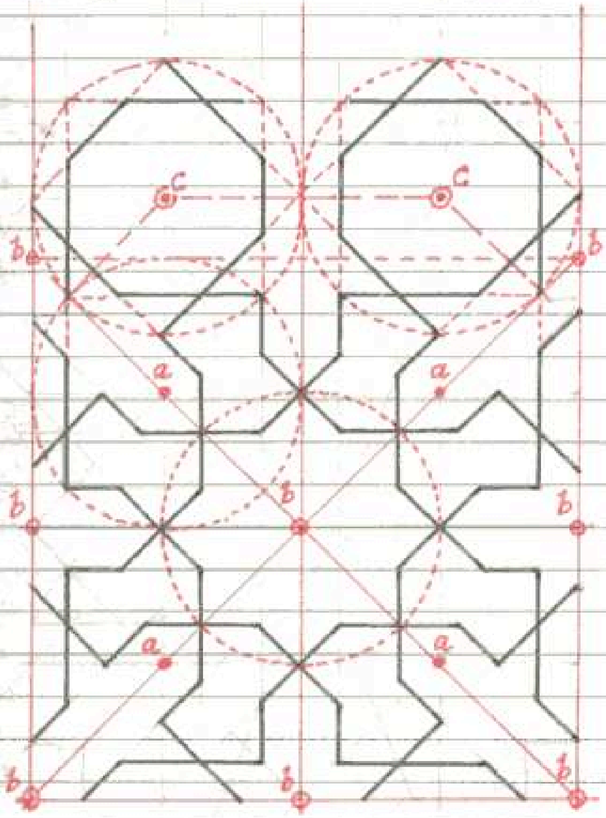
Wed 30 May 1984



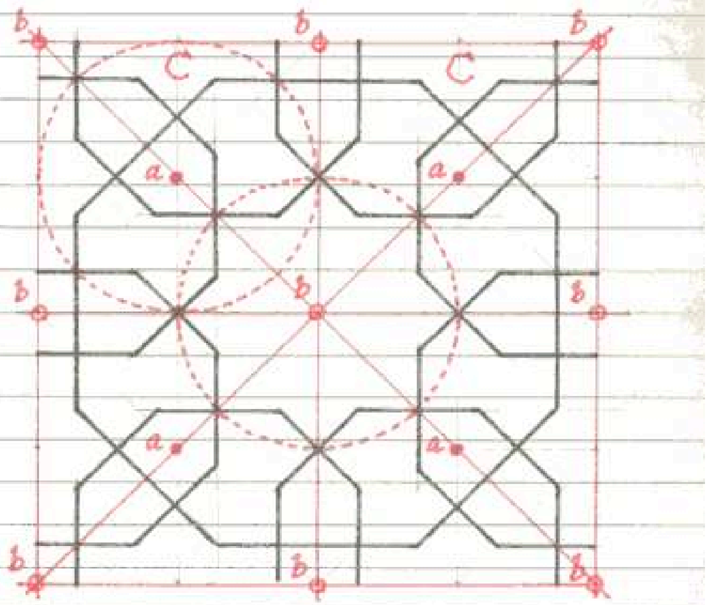
A. Star (1) and Cross (2) pattern
early 8th cent. onwards (Syria)



B. Tomb of Nāṣir bin 'Ali, Uzgend (1012-13 A.



D. Façade of Maghak-i Attari mosque, Bukhara, early 12th century. A derivative of C.



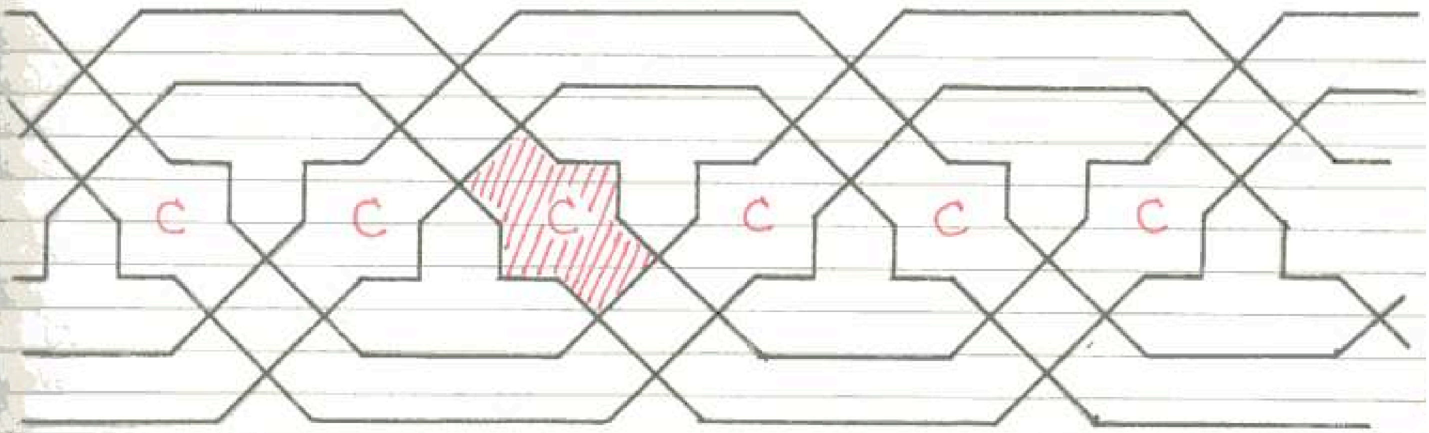
C. A common early derivative of pattern
First appearance c. 1050 – 1100 A.D., cent
Asia.

After Wed 30 May 1984

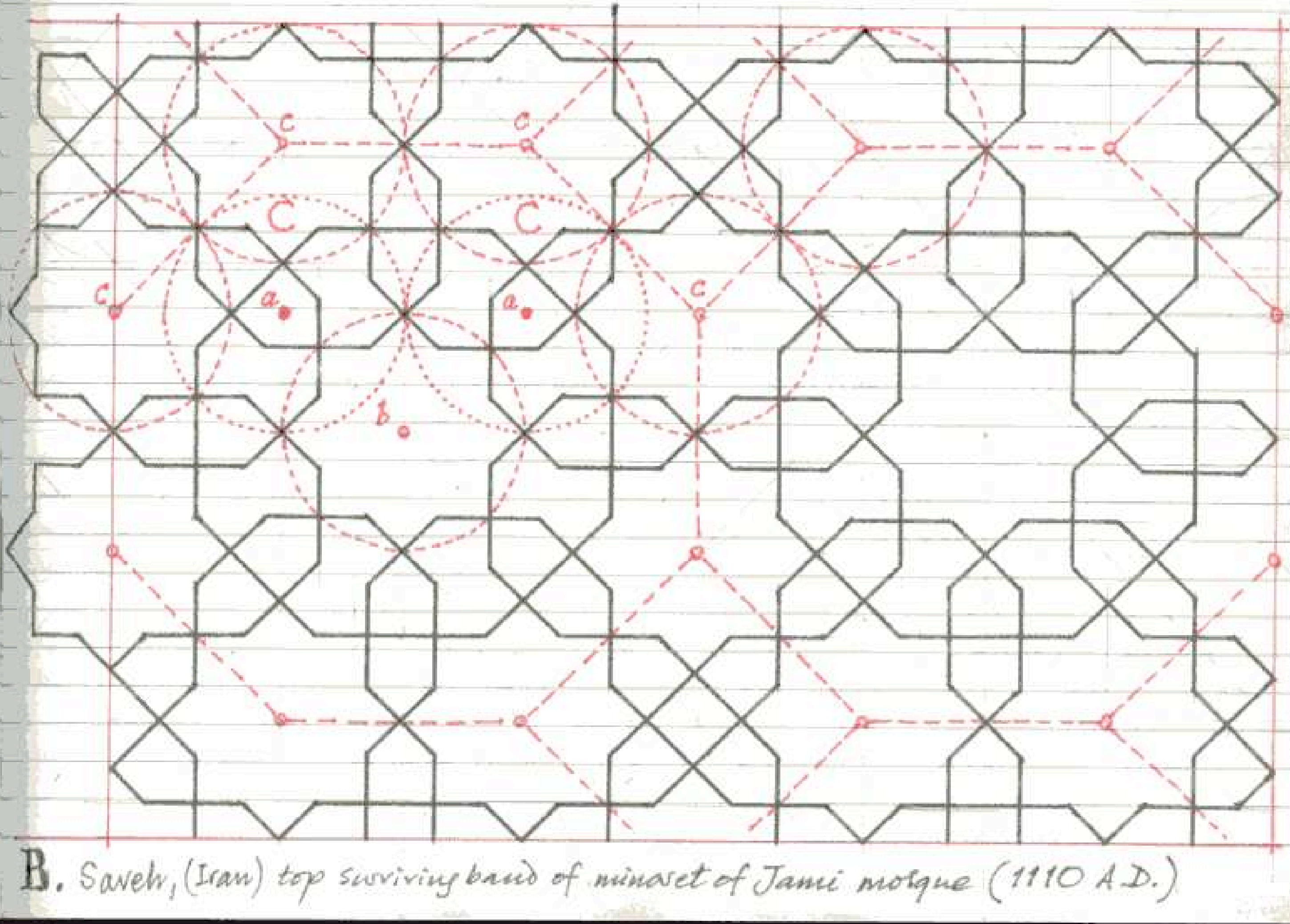
curvilinear version of the inverse star & cross occurs on one of the window grilles at Qasr al-Hair al-Gharbi (see the rough sketch on p.155), dated 727 A.D. Rectilinear versions were probably in existence no later than around 1000 A.D., but no really early examples appear to have survived(?)* However, we should not ignore the possibility of further corroborating, if indirect evidence given by patterns such as that of fig. 170 A from the late 11th century. This contains certain areas, labelled C in the drawing, which are characteristic elements in the inverse star & cross pattern, and which might therefore be taken as evidence that the ornamental band was built around a template derived from an oblique band selected from the inverse star & cross.

Another early pattern is a derivative of fig. 168C, in which the border areas C are seen as parts of eight, mainly obscured (8/2)1 - stars identical to that at the centre of the repeat square, point b. When thus completed (fig. 170 B) we arrive at a pattern in which 8-stars are centred on the vertices and octagonal centres of the semi-regular tessellation t{4,4}, outlined in red interrupted lines in the drawing opposite. The earliest surviving example of this pattern seems to be that round the top of the remaining stump of the minaret of the Jami mosque, at Saveh (Iran) dated 1110 A.D. This pattern is thus close in time and place to most of the earliest examples of the pattern of fig. 168 C, and it may be that it was in fact derived from the latter in the manner suggested. Note that the pattern of fig. 168D, from the early 12th century façade of the Maghaki Attari mosque, Bukhara, makes a similar observation, but here the elements centred on points c are regular octagons which can be inscribed on the inner points of the 8-stars. Numerous later variations on the Bukhara pattern are known; among Bourgoin's (1879) collection we may cite his plates 52, 55 and 64. The star-centred octagon pattern (fig. 168C) is shown in Bourgoin's plate 67.

* One not particularly early example occurs on the earlier Kharraqan tomb tower (1067); see S.P. & H.C. Seherr-Thoss, 1968 Plate 21



A. Band of brick ornament round lower half of Kirat minaret, Iran (late 11th century). This includes shapes C, characteristic of the inverse star & cross pattern, and may indicate its derivation from, and therefore the prior existence of, the inverse pattern.



B. Saveh, (Iran) top surviving band of minaret of Jami mosque (1110 A.D.)

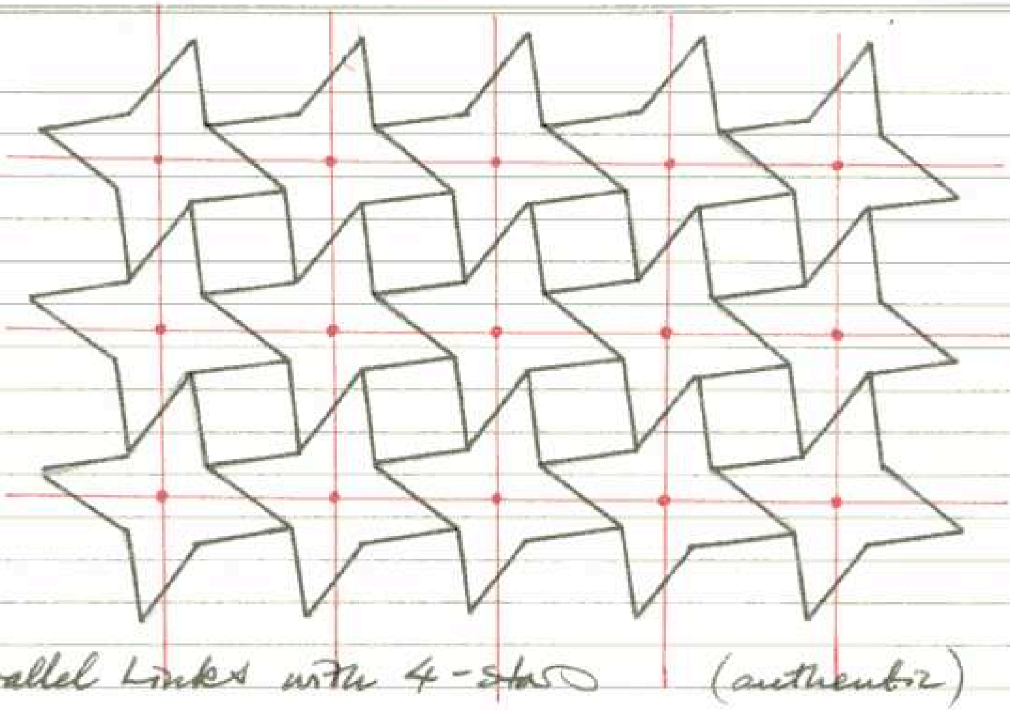
Wed 30 May 1984

It is of some importance to date the earliest examples of derivative patterns of this kind (and it is no less important to recognize them as derivative patterns, of course) since they may form evidence of a general surge in inventiveness and insight among geometrical pattern designers during the 10th century to the early 11th century. Grabar (1978, "The Alhambra", pp. 195-6) refers to recently discovered, as yet unpublished evidence that mathematicians and scientists were responsible for guiding artisans towards the discovery of new methods of composing patterns with stars and polygons after the 10th century. Rogers (1973, "The 11th century, — a turning point in the architecture of the mashriq?" pp. 221 ff) discusses the early appearance of rectilinear star patterns in Central Asia, and considers that the crucial period of its development seems to be, from a number of examples he quotes, in Khurasan, during the late 11th century.

From the few patterns just discussed in these notes, it is arguable that new methods of composing rectilinear star patterns began to be applied to the star and cross pattern at least, no later than the beginning of the 11th century. Of course, the method itself may not have been newly evolved, but may have been used, even if unconsciously, in the discovery of new designs involving non-rectilinear or even non-geometrical ornament.

In discussions of topics along these lines there is a great need for accuracy in defining just exactly which kind of ornament we are discussing, which motifs, and exactly which processes of design and invention we are trying to pinpoint. In the suggested discovery of new patterns from an initial star and cross basis several distinct methods of derivation are involved; it is intended that these shall be catalogued and distinguished shortly in this notebook, but let us first trace a few more examples of similar early pattern derivation.

Thu 31 May 1984



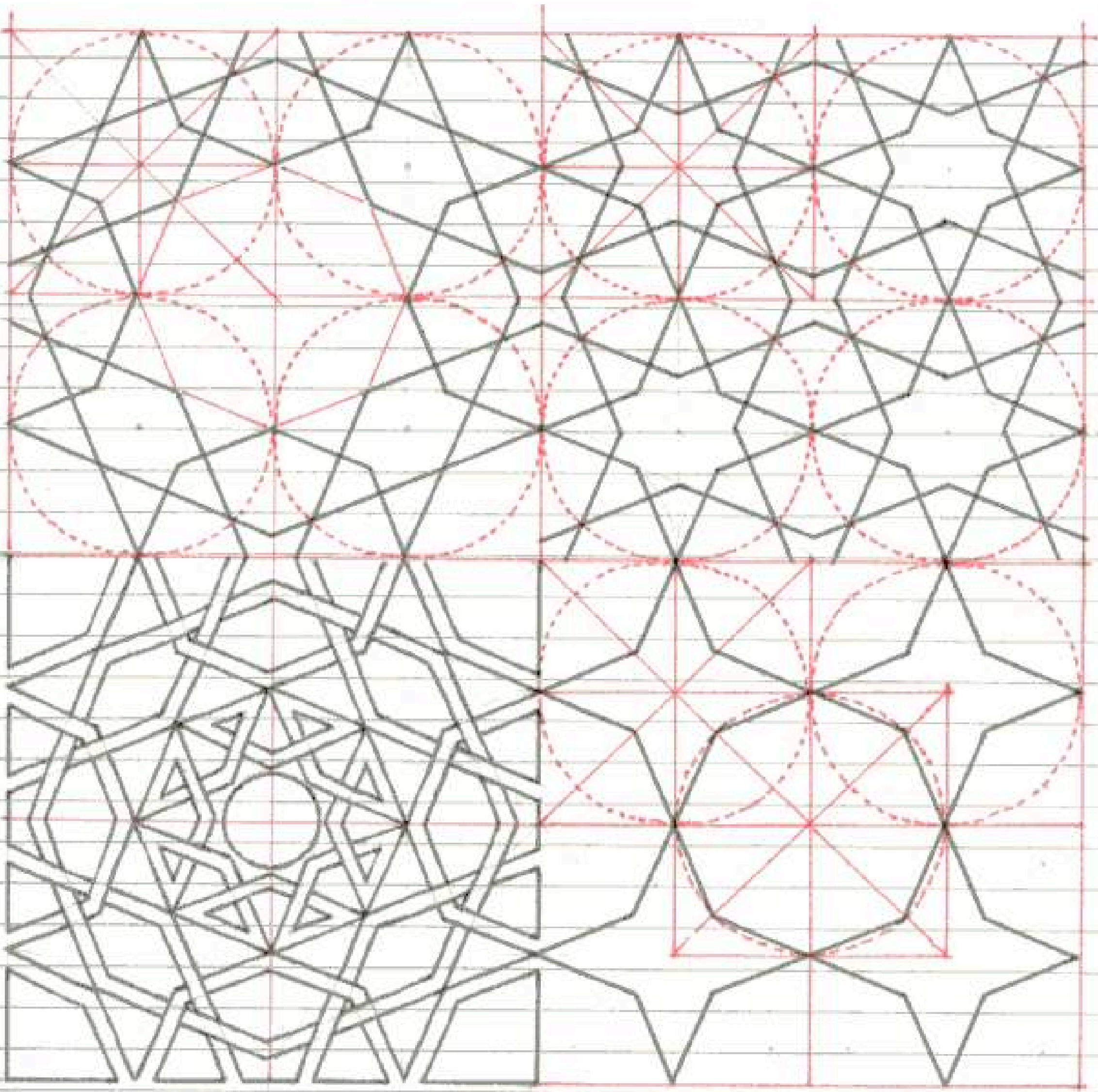
A

Parallel Links with 4-stars (authentic)

A Pur Thu 31 May 1984

In spite of some inaccuracy in the execution of the original, the scheme shown in fig. 174 A, B is undoubtedly the intention of the designer of the middle pattern on each vertical strip beside the entrance to the Maghak-i Attari mosque in Bukhara. It is executed in non-interlacing straps, and the interstices are mostly filled with floral decoration. Note that the straps do not straddle the constructed lines, but are widened on one side only of the lines, that is on the side towards the centres of all octagons. Apart from the subsidiary motifs centred in three large octagons, all constructed lines consist of closed loops which are regular octagons, of two sizes.

The pattern is superposable on the arrangement of regular octagons and 4-stars in fig. 174 D and might therefore be regarded as derivative of that pattern, by addition. However, it is also superposable on the pattern of 8-stars in fig. 174C, and this pattern contains all the main lines of the Bukhara design, so it could be regarded as the parent design, leading to the Bukhara pattern by subtraction. Pattern D is contained in pattern C, that is, the one is a subset of the other. The original designers of pattern C would almost certainly have been aware of this relationship, but I do not think it is likely that pattern C was originally discovered by superimposed identical 4-stars on those of pattern D, but rotated 45°. This is actually the manner in which many writers have supposed 8-, 10- and 12-stars to have been discovered, although it is not clear why they should imagine that the original designers were unable to construct the larger stars directly from 8-, 10- or 12-fold divisions of a circle. The 4-star and octagon pattern was certainly in existence long before Islam, going back to classical times. The parallels between the Bukhara pattern and the 8-star pattern of fig C make it seem likely that pattern C was the origin of the former, and if so this forms strong evidence for the prior existence of pattern C at the beginning of the 12th century. Other indirect evidence also



A C

B D

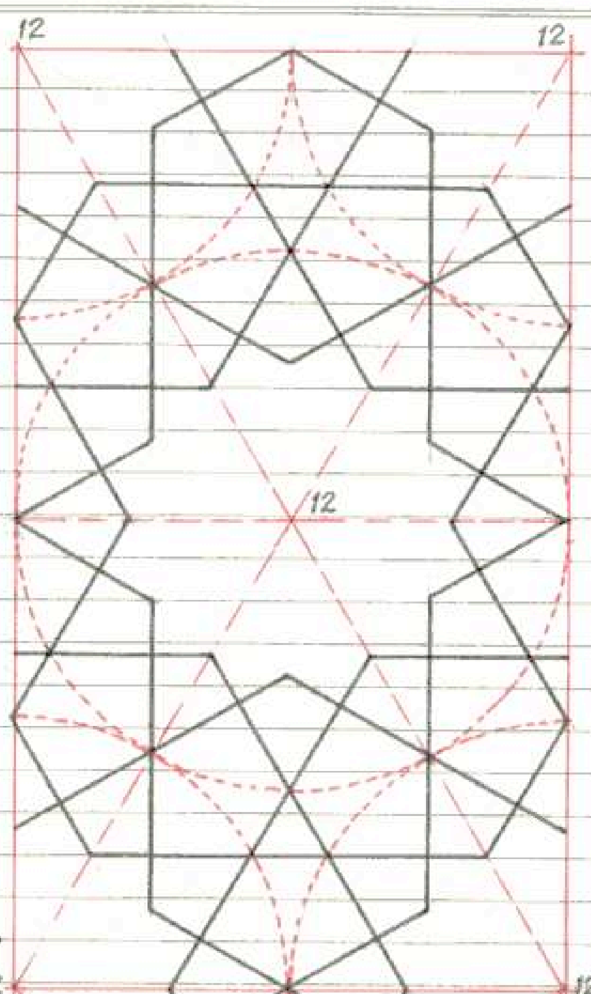
A, B from façade of Maghak-i Attari mosque, Bukhara (early 12th cent.) See Hill & Grabar, "Islamic Architecture..." figs 3-8, especially fig. 5.

points to this conclusion. Pattern C above becomes, in my notation (8/3)2-segment stars in Sp1(2×2)8,8. A pattern of (12/4)2 on a triangular grid occurs on the earlier of two Kharagan tomb towers (1067–68 A.D.: Stronach & Young 1966 Plate IXa). Now it seems almost certainly generally true that lower numbered stars, alone or in repeating patterns, were discovered and used as decoration before higher numbered stars. So we might expect pattern C to have been discovered before 1067 A.D., even though no early examples of it seem to have survived.* For similar reasons it seems probable that Rp1(3x2)10,10/I was also known at the time the earlier Kharraqan tomb tower was built, but the first example of its use seems to be on a tympanum in the North dome chamber of the masjid-i Jami, Isfahan (1088 A.D.), twenty years later.

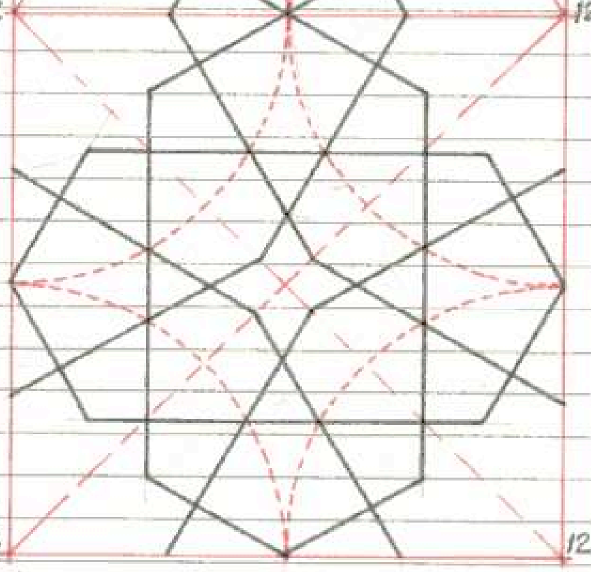
All this evidence points to a surge of pattern designing activity during the 11th century in this general area, thus agreeing with the views expressed by other authors, especially Rogers (1973, referred to on p.171), that the 11th century was a turning point in the art of eastern Islam. Rogers suggested the late 11th century, but I think the heightened activity in the discovery and design of new star patterns probably began in the first half of the 11th century. However, in spite of the statement of Grabar (1978, referred to on p.171) I cannot see that it is at all necessary to assume that this increased activity was the direct result of the intervention of professional mathematicians. After all, before this period artisans were quite capable of drawing accurately constructed geometrical patterns involving the use of square or triangular grids, and of drawing circles, and regularly placed points on the circumferences of these circles, to be joined up in various complex ways to produce regular polygons and stars. It therefore seems entirely possible that the artisans could have made the discovery of the new patterns which appeared during the 11th century entirely by themselves, even though the process may have been speeded up somewhat by the intervention of mathematicians. But the principles involved in the construction and invention of the new patterns were merely an extension of the methods with which the artisans were already familiar.

Fig 176 (opposite) The four patterns shown were probably discovered near to one another in time no later than about 1050 – 1070 A.D. →

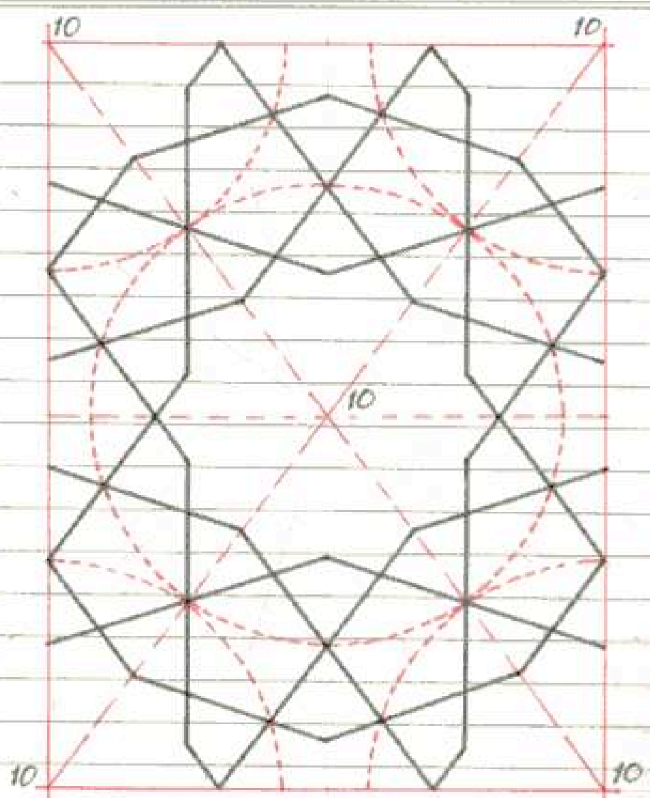
* the (8/3)2 star occurs alone, inscribed on the vertices of an octagon, on the Damavand tomb tower, thought to date from soon after 1050 A.D. (S.P. & H.C. Scherr-Thoss, 1968, Plate 8)



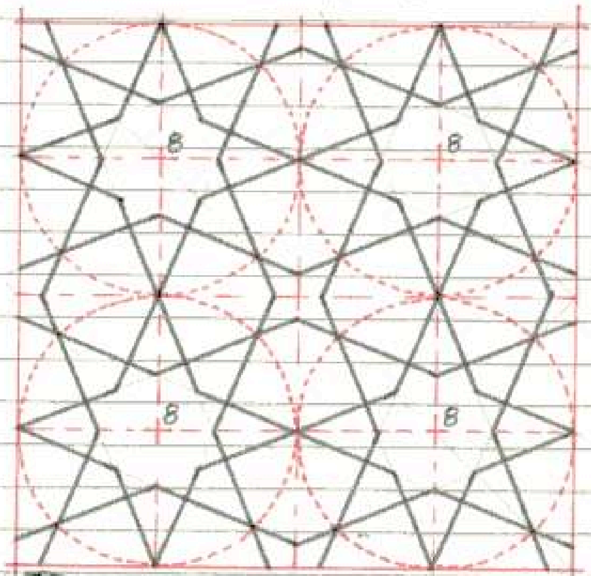
A.



B.



C. (10/3)2-stars on Rp1(3×2)10,10
Earliest(?) surviving example: 1088 A.D.
North Dome Ch. masjid-i jami, Isfahan



A: (12/4)2-stars on H1(4×2)12,12. Earliest(?) surviving example: 1067 A.D. First Kharragan tomb tower

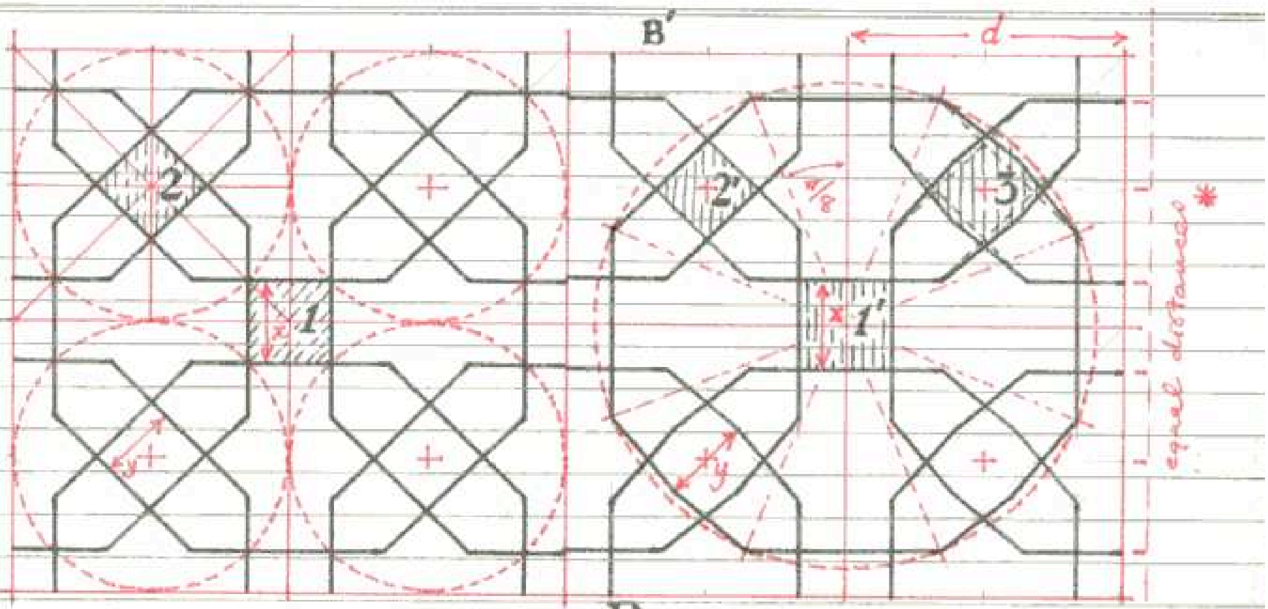
D. (8/3)2-stars on Sp1(2×2)8,8. Implied(?) on façade of Maghak-i Attari mosque, so assumed to have been known about 1100 at the latest. (8/3)2 stars on Demavand tomb tower (soon after 1050)

Sun 3 June 1984

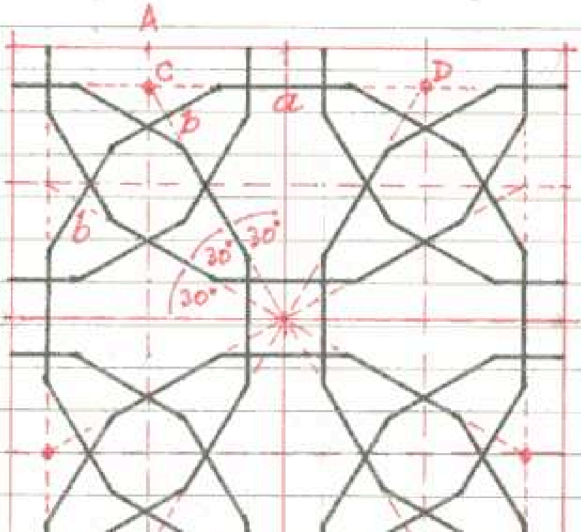
The lowest of the three patterns on each side of the entrance façade of the Maghak-i Attari mosque, Bukhara, present a number of curious features. From a casual glance they appear to be examples of the inverse star-and-cross pattern (fig. 178A), but closer inspection shows that the interpenetrant regular octagons of the latter have become non-regular dodecagons due to the bending of four sides of the octagon into a pair of line segments each. The patterns on each side of the mosque façade are not quite identical — the pattern on the right side seems to have almost regular dodecagons, but measurement of photographs reveals that the sides are not all the same length; the pattern on the left is in fact little different in appearance from the theoretical construction of the inverse star-and-cross pattern, but the square at the centre of each original "cross" has here become an obtuse angled, non-regular octagon (fig. 178C, B respectively).

It is not immediately obvious why these modifications have been made, but fig. B seems possibly an intermediate stage towards the nearly regular dodecagons of fig. C. In the theoretically exact construction of the star & cross inverse (fig. 178A) two kinds of small squares are outlined, labelled 1 and 2 in the figure. These arise as residual spaces between four octagons in each case, but the first as the space remaining <u>outside</u> the octagons, the second as the space common to four <u>overlapping</u> octagons. Theoretically squares 1 and 2 are equal (see fig 180A, B), that is, distances x and y in fig. 178A are equal. This equality is most easily achieved if the pattern is built up on the basis of either fig 178A or 180A and the large overlapping octagons left to take care of themselves. If the pattern construction starts by drawing the large octagons it is possible that small errors in their size could occur; now, the relative sizes of the small shaded squares are sensitive to small variations in the sizes of the overlapping octagons — the size of one will be increased, the other decreased, and the difference in size will be <u>twice</u> the error in linear size of the octagons.

Monday, AUGUST 1, 1966



A. Star & cross inverse, theoretical pattern. B. Maghak-i Attari mosque, lowest panel to left of entrance.



C. Maghak-i Attari mosque, Bukhara. Lowest panel to right of entrance.

On the original side a of the dodecagon is rather longer than side b, so the polygons are not regular.

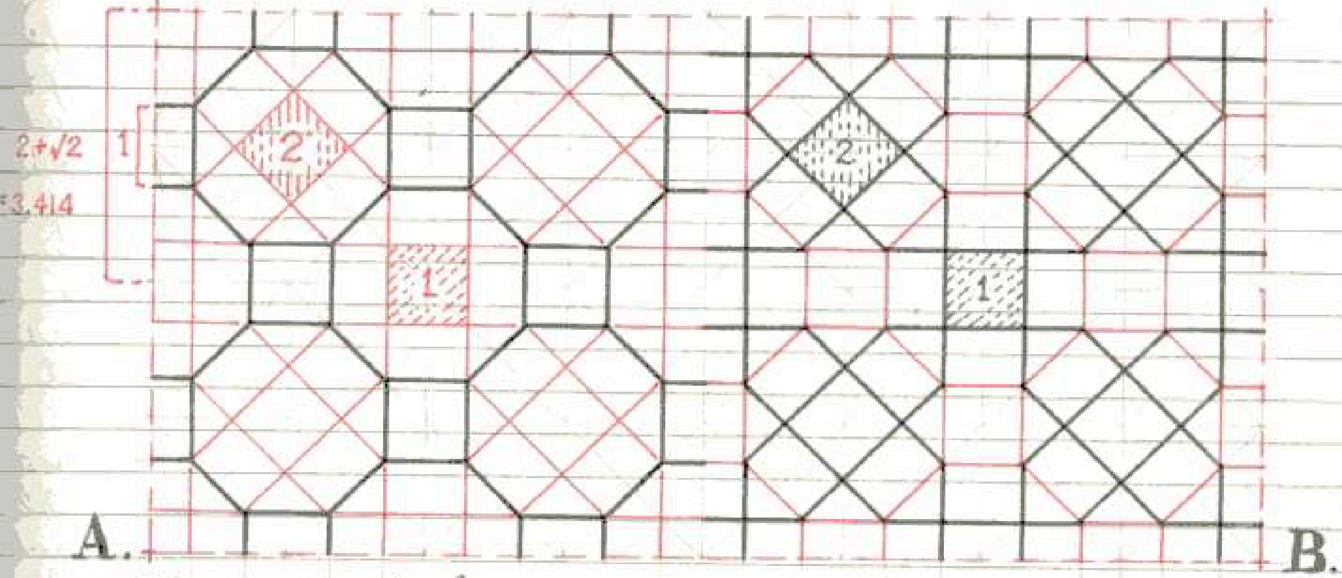
These patterns on the entrance façade of the Maghak-i Attari mosque are so roughly laid out that it is difficult to be certain of the exact method of construction of some of them, particularly the two shown here, figs. B and C. However, measurements on published photographs in Hill & Grabar (1964) and in Kemper (1961, fig. 68-2) makes the present reconstructions plausible. It seems likely that the interlacing bands were widened on both sides of the constructed pattern lines to an equal amount - which is not so on the middle pattern (see fig. 174)

* measurements on published photos show that d/x is very nearly 3. In the inverse star & cross this ratio is equal to $2+\sqrt{2} = 3.414$ (see fig. 180A)

Mon 4 June 1984

In the pattern on the left of the mosque façade, measurements on photographs indicate that square 1 is slightly larger than its theoretical size (cf. 178 A and B). This would make square 2 smaller than square 1, as shown at the upper left of fig. 178B, and the difference is quite noticeable, even though the initial error differs from the theoretically exact version by such a small amount. This difference in size is effectively masked however, since each square 2 is expanded slightly, as shown at position 3 on fig. 178B, into a non-regular octagon. Thus the visual effect is that the space enclosed by the outlines of square 1' and octagon 3 appear more nearly equal. It is course by no means certain that this slight modification, or adjustment, was originally carried out to mask a small error, but the possibility, and its manner of execution, form an intriguing question. Perhaps the sculpting out of square 1 had proceeded before the error was noticed, which would have made it impossible to alter the size of the overlapping octagons. This would presuppose that someone was aware that squares 1 and 2 ought to be equal in size when the inverse star and cross pattern is properly constructed.

However, even supposing that the peculiarities of the left side pattern are to be explained as an attempt at masking a small error in construction, this does not explain the choice of a quite different (if topologically identical) pattern for the corresponding right hand panel. These lowest panels are at eye level, and if someone were concerned that an error on the left side pattern might be noticeable it seems strange that he should so to speak draw attention to the fact by substituting a different pattern at the same level on the right of the entrance façade. Whatever the answer, it seems that we are witnessing here a case in which either the pattern designer or the artisan executing the finished product failed to understand the geometry of whatever pattern he was trying to construct.



The inverse star & cross can be constructed on the basis of the semiregular tessellation $t\{4,4\}$, in the octagons of which four of the O3 diagonals are drawn, so as to include a central square, but in two different orientations, labelled 1 and 2 above. It will thus be obvious that the small shaded squares at positions 1 and 2 are equal (cf. fig. 178A), but rotated 45° with respect to one another. It is also obvious that the large overlapping octagons of the completed pattern (black lines in fig. B) are regular, since each side is one O3 diagonal of an octagon in the original semiregular tessellation.

181 Summary of fundamental notions
Star-centres & Star-motifs.

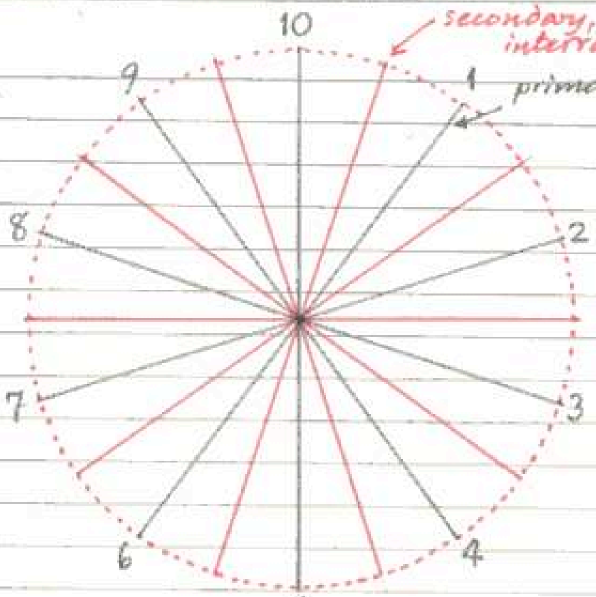
Fri 8 June 1984

In their fundamental form the star-motifs employed in Islamic geometrical patterns almost invariably belong to the dihedral point groups (D_n), in which n-fold rotational symmetry is combined with the presence of n mirror axes intersecting at the centre of symmetry (fig. 182 A, C). Only rarely do these motifs lack mirror axes (cf. p. 56) and therefore pertain to the cyclical point groups (C_n), possessing only rotational symmetry. Even in these cases the motif usually has basically dihedral point symmetry, but the addition of asymmetric elements destroys the original mirror axes. However, there is one very common method by means of which a fundamentally dihedral star motif is transformed into one with purely rotational symmetry, and that is through the substitution of its skeletal line segments by interlacing bands (fig. 182 D). This gives a handedness to the motif, which can therefore exist in either of two enantiomorphic varieties. The same applies to a periodic pattern containing star-motifs in which the pattern lines are represented as interlacing bands, but the fact is sometimes overlooked.

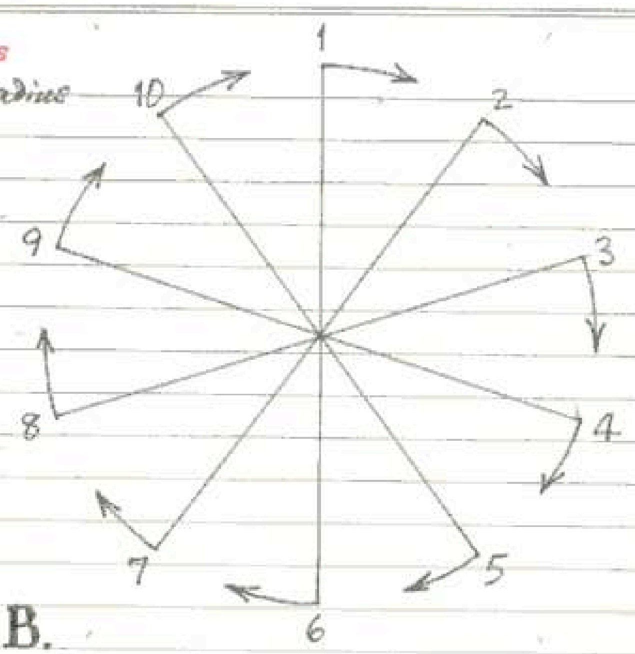
An analysis of a body of ornament by means of the 17 plane groups (and if necessary making use of the point groups and the frieze groups also) is only of interest at the most fundamental level, but in studying a specific set of patterns such as the Islamic star patterns a much finer classification becomes necessary, to distinguish between many different patterns and styles sharing a single underlying symmetry group.

In forming periodic patterns from star-motifs the latter are almost invariably linked in certain geometrically defined ways, as outlined above (pp. 157 etc). In forming a periodic plane pattern any given star-motif will form links with at least three neighbouring motifs, usually more. An extended system of such links covers the whole plane forming a net or tiling of the plane in which the star-motifs are centred on the

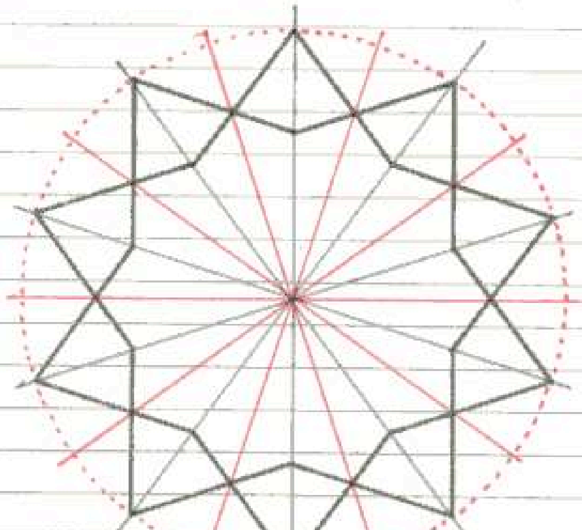
Friday, AUGUST 5, 1966



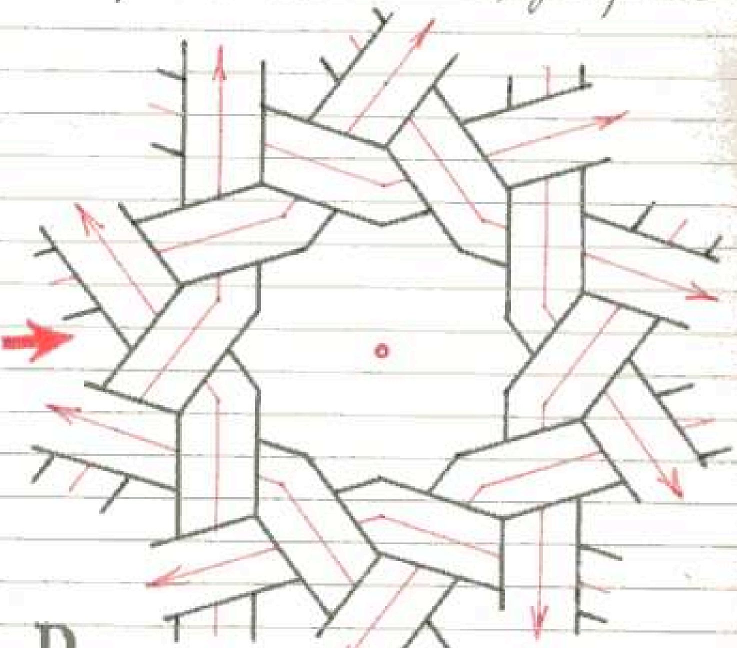
A.
Dihedral Star-centre*, group D_{10}



B.
Cyclical Star-centre, group C_{10}



C.
Dihedral Star-motif, group D_{10}
type (10/3)2.



D
Cyclical Star-motif, group C_{10},
type (10/3)2.

* or, geometrically speaking, this might be termed a "star" – the basis for every "star-motif"; the latter being an actual ornamental realization which can be used in a definitive star pattern.

Fri 8 June 1984

vertices and the centre-to-centre links constitute the edges of the tiling. Since the links themselves are the straight lines between pairs of neighbouring star-centres – or between the centres of pairs of neighbouring star-motifs – each cell of the tiling will be an ordinary, rectilinear polygon, but not in general a regular polygon. If the star-motifs are regularly constructed the interior angles of each cell of this tiling will be some integral multiple of 180°/n, where n represents the number of primary radii in the star-motif at any vertex under consideration. Let us represent the star-numbers of the motifs centred on the vertices of any cell of the tiling, in successive order round the circumference of the cell, by the series of letters ..... K, L, M, N. Let us then represent the numbers of divisions of the fundamental interradial angle 180°/n at successive internal angles of the same cell by the letters P, Q, R, S..... Thus each cell may be characterised in a similar manner to that of the integral kites and rhombs on pp. 89 – 93 by the notation

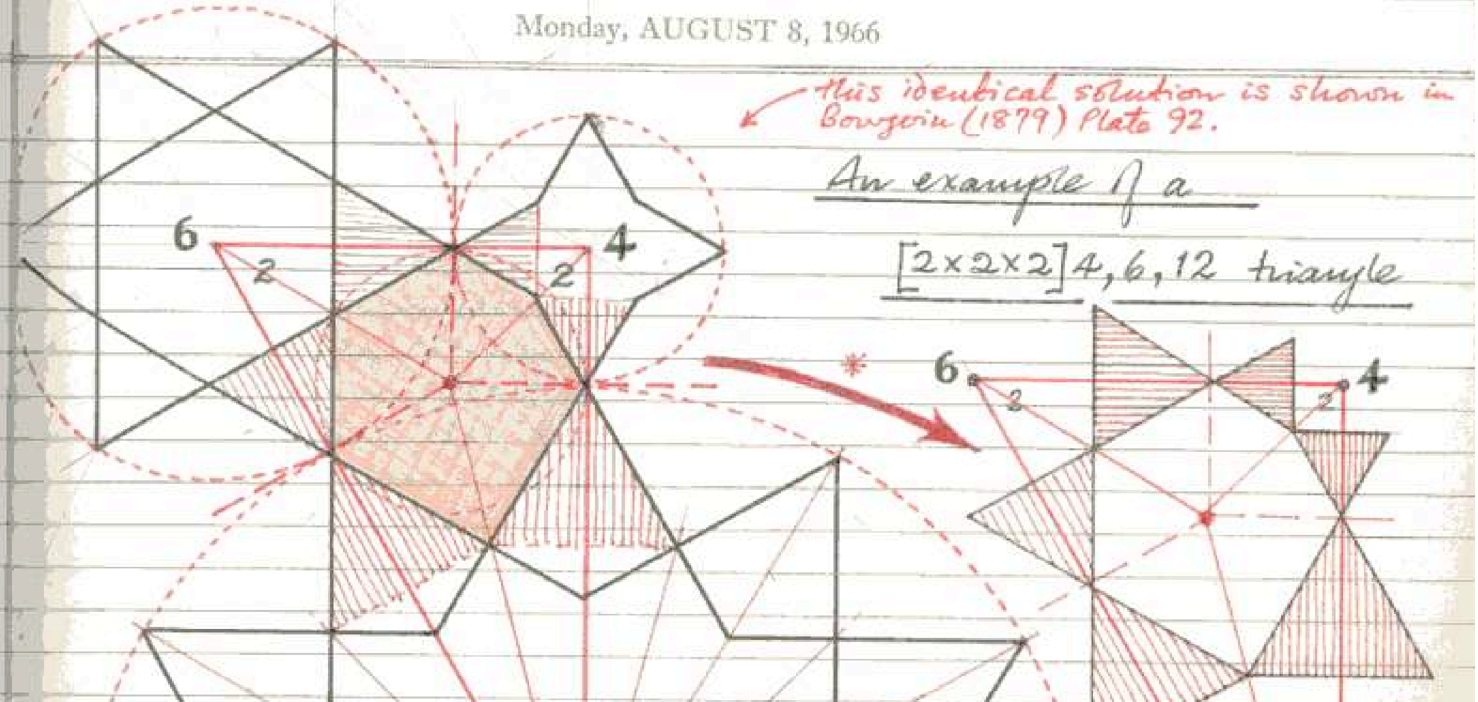
$$[P \times Q \times R \times S \times \ldots] \ldots K, L, M, N.$$

This notation (which of course can take different forms) precisely defines the angle sizes within the cell, which are given as fractions of 180° by the expressions

P/K, Q/L, R/M, S/N etc. However, the exact shape of the cell is not necessarily defined by this means! If the cell has axes of symmetry and the number of kinds of star-motifs is fewer than the number of vertices in the cell, it may of course be possible to simplify this notation considerably, as in the case of rhombuses considered above (pp. 89, 90).

When all motifs in a pattern are of the same type and have the same star number they can all be of one size, but if two or more kinds (i.e. star-numbers) are present simultaneously, trial and error will soon this identical solution is shown in Bourgoin (1879) Plate 92.

An example of a [2×2×2] 4, 6, 12 triangle



* The shaded triangles are all similar (in this case all are equilateral)

The relative lengths of the three circumradii are uniquely determined, but not the exact slopes of the lines.

In triangles of this kind, [2×2×2] in which pairs of star-motifs are linked along all three sides by means of primary radii, the secondary radii from each vertex, each bisecting its respective interior angle of the triangle, all meet at a single point which is the in-centre of the triangle. Each star is inscribed in a regular polygon, and the three polygons share a common edge length and meet in a three-way node at the in-centre of the triangle. In this way, the relative circumradii of the three stars are determined. It is obvious that the three kinds of shaded triangles emphasized above are always similar triangles.

Mon 11 June 1984

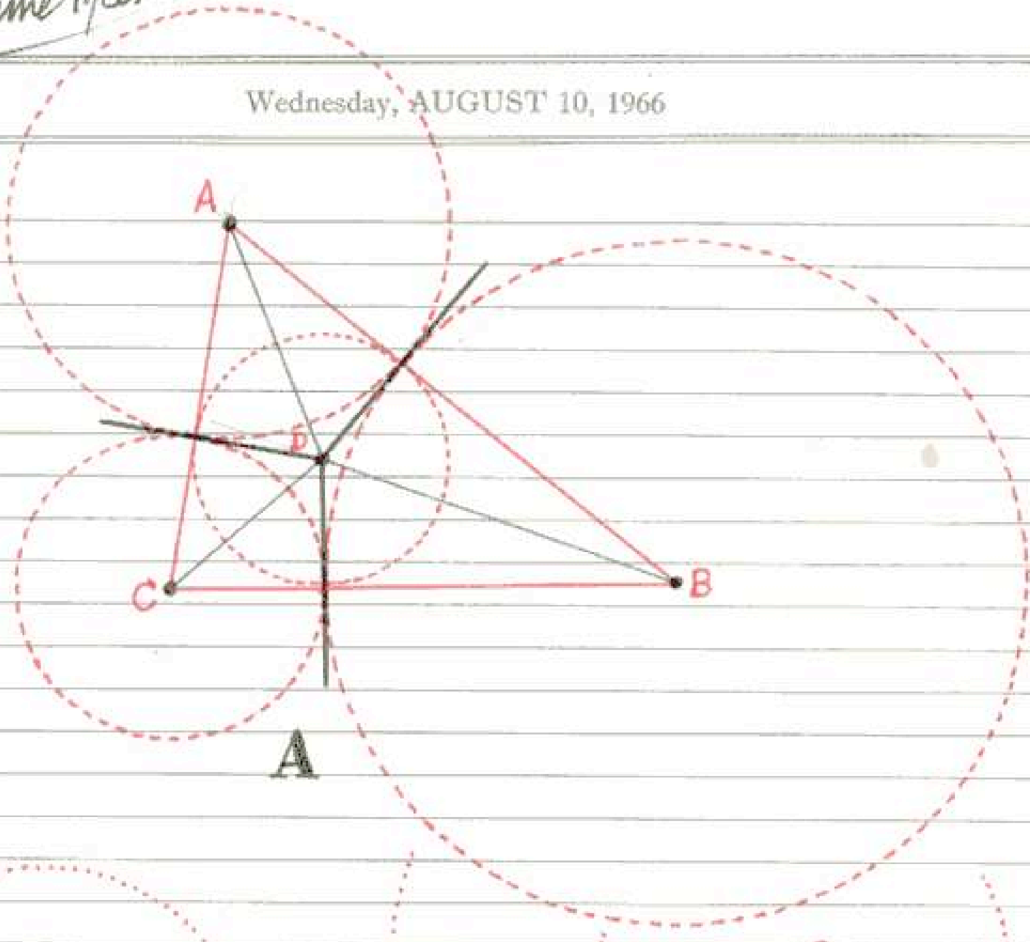
reveal the fact that better results are obtained if the star-motifs have different circumradii, although it is not immediately obvious whether there is a definite ratio between the lengths of the two circumradii of a given collinear link, nor how to construct this exact ratio. The degree to which a definite, or indeed fixed, ratio can be justified mathematically, is variable.

If we choose any triangle (fig. 186, triangle ABC) and centre circles on the vertices, of such sizes that they are tangent to one another, then the relative radii of the circles are completely determined for that triangle. Perpendiculars from the incentre (D) of the triangle to each side give the points at which each pair of circles is tangent. The incentre is also the meeting point of the bisectors of the three interior angles of the triangle. Thus, if we wish to inscribe star-motifs in these three circles so that their outer points coincide on each side of the triangle, then obviously the sizes of the motifs are completely determined ~~and~~ independently of the number of points each has. The number of π/n divisions at each interior angle of the triangle must obviously be even in this case, and the simplest example is therefore the [2×2×2] triangle shown in fig. A opposite.

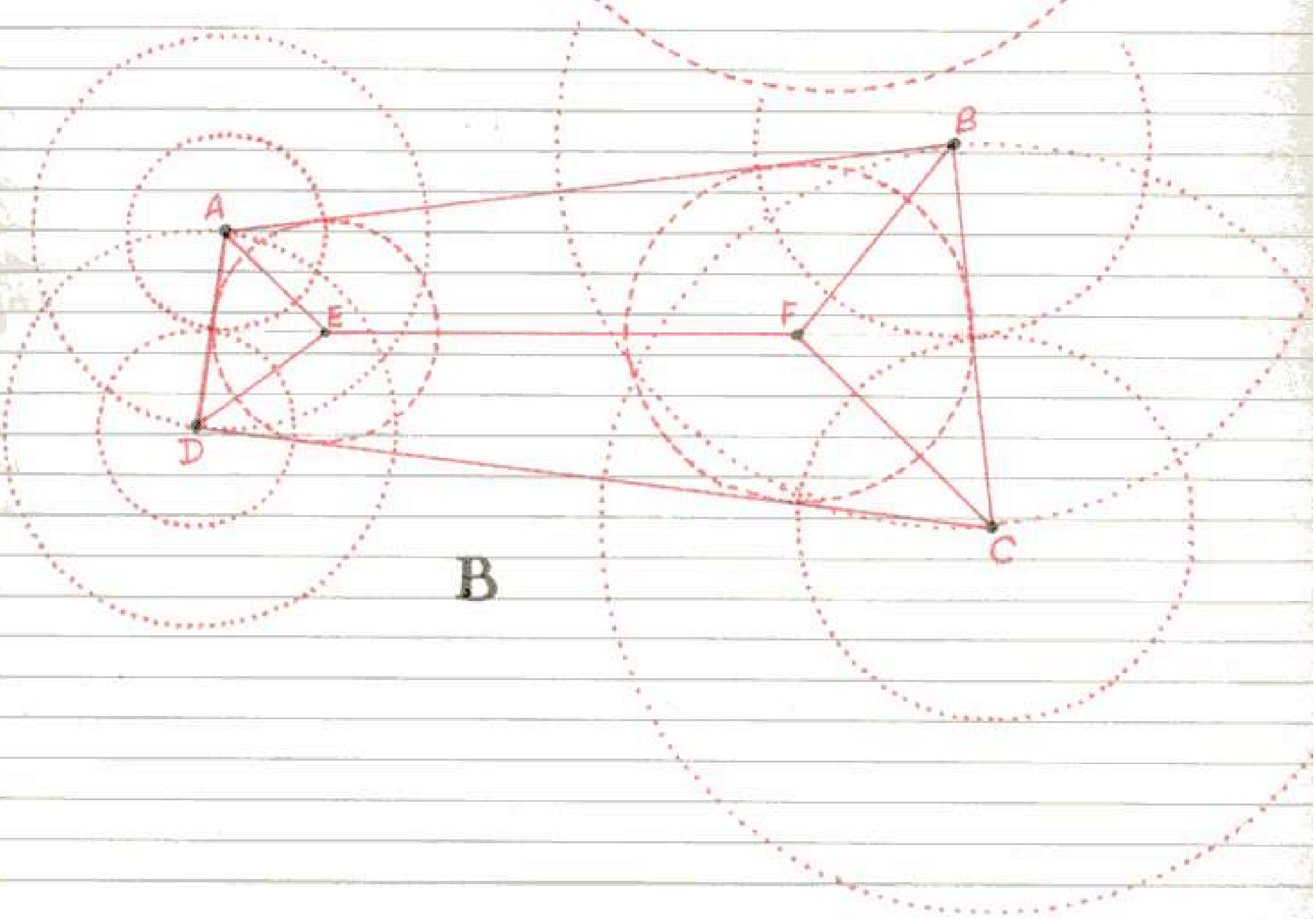
For any arbitrarily chosen n-gon for $n > 3$ it will usually be found that it is not possible to centre n circles on the vertices so that they are tangent to one another in pairs along the edges of the n-gon (fig. 186B). If it is to be possible to do this, then the length of each side of the n-gon must be less than the sum of the lengths of the two adjacent sides.

In order to form star-motifs on the vertices of any n-gon it is the sizes of the interior angles which is important, rather than the relative lengths of the sides. By adjusting the relative lengths of the edges of any n-gon it is possible to construct

Mon 11 June 1984



A



B

Mon 11 June 1984

an n-gon with the same cyclical order of angle sizes which touches a single incircle at every edge. Again, the number of π/n divisions at each interior angle of the n-gon (the n's refer to different things here) must be even, if we require that the links be of the same type along each edge (see p. 157). The simplest example will again be that in which each interior angle has two divisions, as in fig. 188 opposite.

In order to find which combinations of star-motifs will fit each kind of n-gon, where each adjacent pair of motifs shares an outer point on one edge of the n-gon, we must discover all integral solutions of such expressions as

$$\frac{P_1}{M_1} + \frac{P_2}{M_2} + \frac{P_3}{M_3} + \frac{P_4}{M_4} + \dots + \frac{P_n}{M_n} = n-2 .$$

P_1, P_2, P_3 etc are what we have referred to above as P, Q, R, etc and $M_1 M_2 M_3$ etc as .... K, L, M, N; n is the number of sides of the n-gon, and the number of tangent circles within each of which we inscribe a star-motif. Thus, for each vertex of the n-gon, P is the number of divisions at that interior angle, M the number of points in the star-motif. Each division at M_n will be π/M in magnitude.

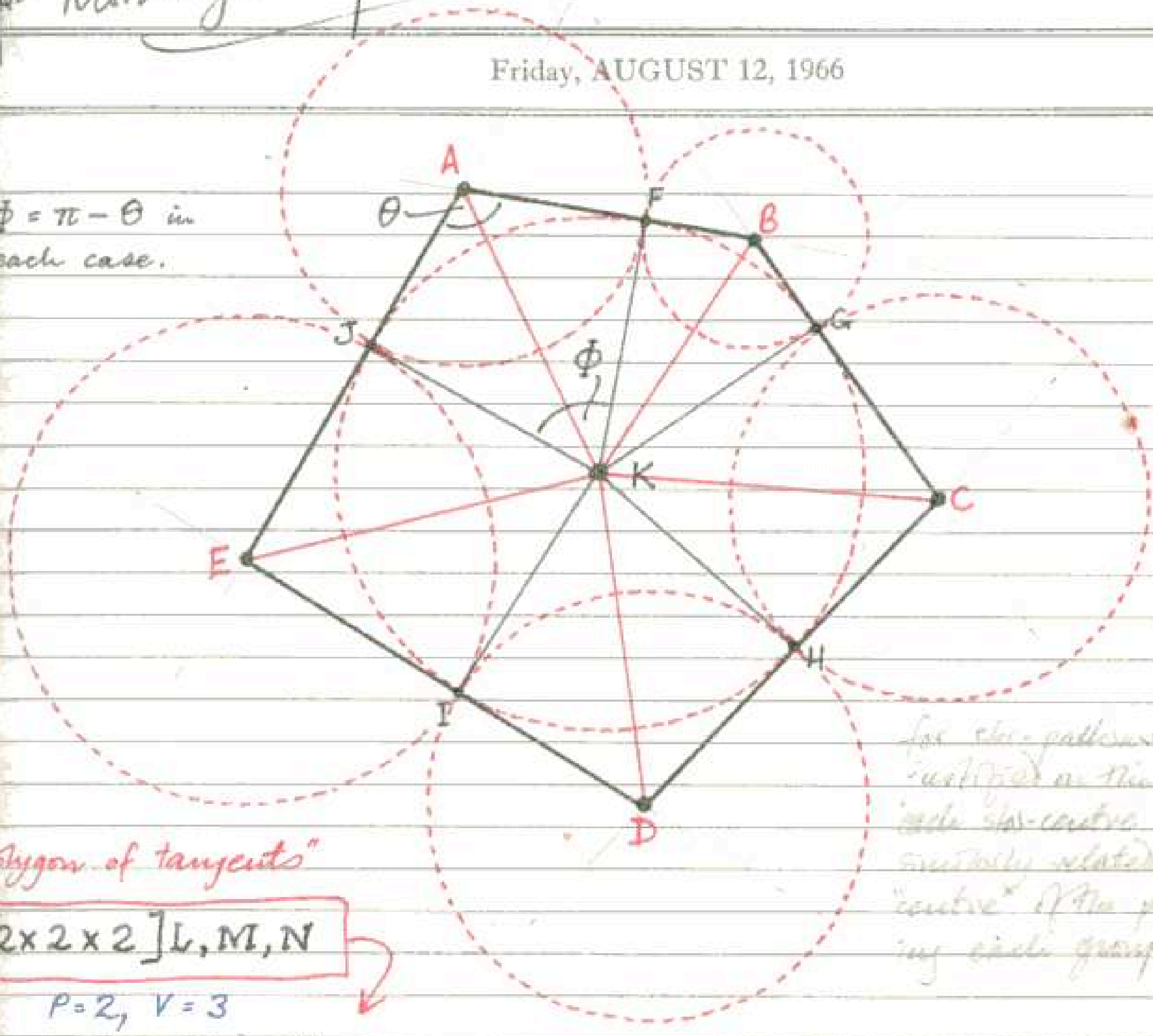
When $P_1\ P_2\ P_3$ etc all equal 2, then the solutions of each equation are equivalent to the different ways in which n regular polygons $M_1\ M_2\ M_3$ etc can meet in an n-way vertex at the incentre of a circle escribed n-gon.

In all cases it is better for the interstitial pattern to have as symmetrical a space as possible, hence the preference for such symmetrical figures as rhombs, kites, etc. as the fundamental building blocks for repeating star patterns.

Apo Mon 11 June 1984

Friday, AUGUST 12, 1966

$\Phi = \pi - \theta$ in each case.



for this pattern, this can be justified on the grounds that each star-centre should be similarly related to a single "centre" of the polygon ... ing each group of star-...

"polygon of tangents"

$\boxed{3[2\times2\times2]L,M,N}$

$P=2,\ V=3$

	M_1	M_2	M_3	
1	3	6	∞	
2	3	7	42	
3	3	8	24	
4	3	9	18	
5	3	10	15	
6	3	12	12 †	*
7	4	4	∞ †	
8	4	5	20	
9	4	6	12	*
10	4	8	8 †	*
11	5	5	10 †	
12	6	6	6 †	*

12 exact solutions

Exact and approximate solutions of

$$\frac{2}{M_1}+\frac{2}{M_2}+\frac{2}{M_3}=1$$

* solutions marked with asterisks are 3-way groupings of regular polygons which can be used in periodic tilings.

† These 5 cases are illustrated on p. 190.

M_1	M_2	M_3	angle sum.
3	11	13	430/429
4	7	9	127/126
5	5	8	21/20
5	6	7	53.5/52.5
5	6	8	59/60
5	6	9	21.5/22.5
5	7	7	34/35

approximate solutions

Tue 26 June 1984

$P = 4,\ V = 3$

27 Integral Solutions of $\frac{4}{M_1} + \frac{4}{M_2} + \frac{4}{M_3} = 1$

	M_1	M_2	M_3	
1	5	20	∞	
2	5	25	100	
3	5	28	70	
4	5	30	60	
5	5	36	45	
6	5	40	40	
7	6	12	∞	x
8	6	14	84	x
9	6	15	60	
10	6	16	48	x
11	6	18	36	x
12	6	20	30	x
13	6	21	28	
14	6	24	24	x
15	7	10	140	
16	7	12	42	
17	7	14	28	
18	8	8	∞	x
19	8	9	72	
20	8	10	40	x
21	8	12	24	x
22	8	16	16	x
23	9	9	36	
24	9	12	18	
25	10	10	20	x
26	10	12	15	
27	12	12	12	x

x = the 12 solutions for 2 (divided by 2)

$P = 6,\ V = 3$

Saturday, AUGUST 13, 1966

60 Integral Solutions of $6/M_1 + 6/M_2 + 6/M_3 = 1$

	M_1	M_2	M_3	
1	7	42	∞	
2	7	43	1806	
3	7	44	924	
4	7	46	483	
5	7	48	336	
6	7	49	294	
7	7	54	189	
8	7	56	168	
9	7	60	140	
10	7	63	126	
11	7	70	105	
12	7	78	91	
13	7	84	84	
14	8	24	∞	x
15	8	25	600	
16	8	26	312	x
17	8	27	216	
18	8	28	168	x
19	8	30	120	x
20	8	32	96	x
21	8	33	88	
22	8	36	72	x
23	8	40	60	x
24	8	42	56	x
25	8	48	48	
26	9	18	∞	
27	9	19	342	
28	9	20	180	
29	9	21	126	
30	9	22	99	
31	9	24	72	
32	9	27	54	
33	9	30	45	
34	9	36	36	

Sunday, AUGUST 14, 1966

	M_1	M_2	M_3	
35	10	15	∞	
36	10	16	240	x
37	10	18	90	x
38	10	20	60	x
39	10	24	40	x
40	10	30	30	x
41	11	14	231	
42	11	15	110	
43	11	22	33	
44	12	12	∞	x
45	12	13	156	
46	12	14	84	x
47	12	15	60	
48	12	16	48	x
49	12	18	36	x
50	12	20	30	x
51	12	21	28	
52	12	24	24	x
53	13	13	78	
54	14	14	42	x
55	14	15	35	
56	14	21	21	
57	15	15	30	
58	15	20	20	
59	16	16	24	x
60	18	18	18	x

right angle

these reduce to the (3×2) [illegible] solutions

x = The 23 solutions for 3 (divided by 2)

Tue 28 June 1984

Thu 7 Feb 1985

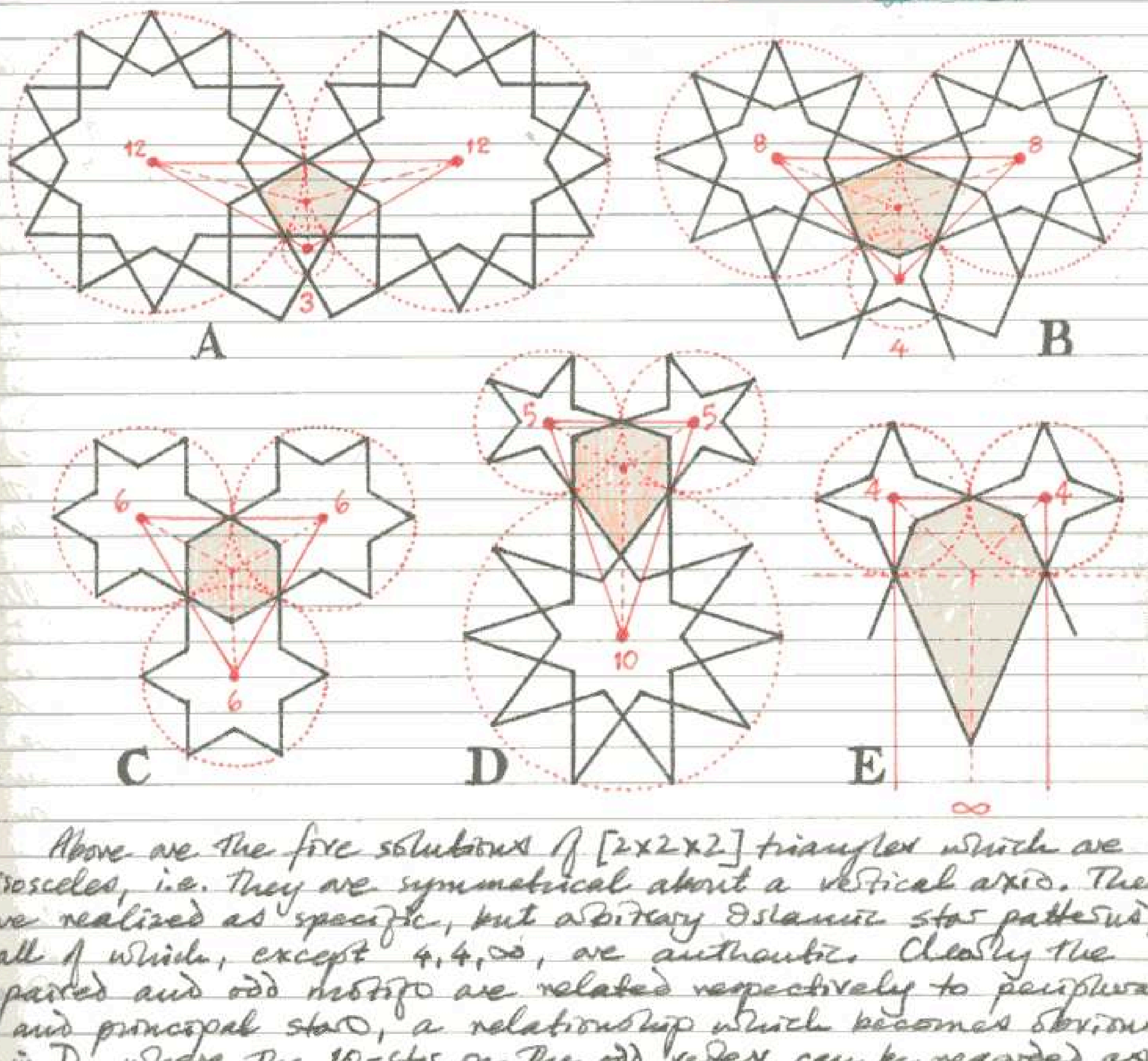
TANGENT STARS ON VERTICES OF [2×2×2] TRIANGLES

Monday, AUGUST 15, 1966

see also p. 75: solutions for (2×1) rhombs.

Type I link between 12-stars:-

Type I 8-stars in a collinear link, but not a type I link:-



Above are the five solutions of [2×2×2] triangles which are isosceles, i.e. they are symmetrical about a vertical axis. They are realized as specific, but arbitrary Islamic star patterns, all of which, except 4,4,∞, are authentic. Clearly the paired and odd motifs are related respectively to peripheral and principal stars, a relationship which becomes obvious in D, where the 10-star on the odd vertex can be regarded as the outer star of a 10-rosette, and the symmetrical hexagon inscribed in the [2×2×2] triangle in an outer cell of the rosette. General relationships between peripheral and principal stars have been dealt with on pp. 19-20, for isosceles triangles.

NB. counted as (p, q) rhombs we have here q = 1, and (2,1) rhombs see p. 14

RPJ Wed 6 Feb 1985

Ambiguities of Type VII Patterns

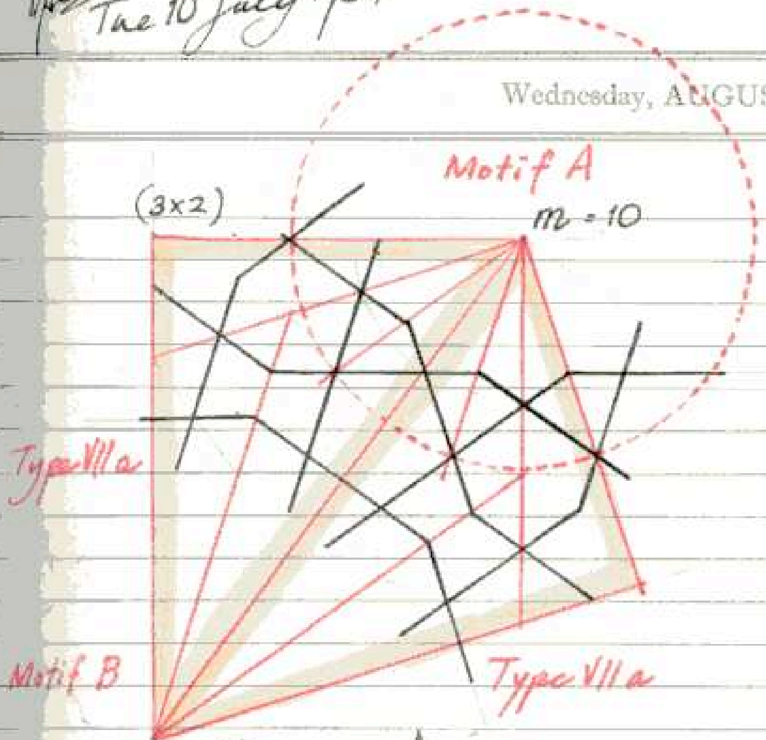
The designation "Type VIII" (previously Type IV) applied to (3x2) or [6x4] rhombs can be ambiguous, since it has sometimes not been made clear whether the "type" referred to motifs, rhombus, or repeating pattern. In fact with regard to motifs the characteristic feature of a Type VII is that it is a "mixed" variety, one motif being identical to those of a Type VIII, while the other is a Type I motif partly realized as a Type III. The difficulty has been that whereas other pattern types, with a single kind of motif, can satisfactorily produce the repeating pattern Rp1(3x2)10,10, this is not so in the case of Type VII. Here, the realization as an Rp1 pattern necessitates two kinds of rhombs — which have been distinguished as VIIa and VIIb (see pp. 83-86 and fig. C on p. 192 opposite). The additional rhomb, type VIIb, results automatically from completion of the existing motifs to fill the spaces between similarly orientated type VIIa rhombs. (similarly for types IX and X which exhibit two different kinds of motif).

Since the Type VIIa variety can give a geometrically exact construction for all (3x2) rhombs when m ≠ n, this must be regarded as the principal variety. However, as we have pointed out already a Type VIIb can be constructed as an approximate version in those cases where m ≠ n, and it does in fact appear in authentic Islamic ornament (in Iran). In the case of the most common (3x2) solutions with dissimilar motifs, namely Sp1(3x2)12,8 and H1(3x2)9,12, the patterns can be achieved with just one variety of rhomb, unlike the central (3x2)10,10. The fact that both a and b varieties occur is probably owing to the fact that the original artisans did not fully understand the geometry of this pattern type (see pp. 33-34, and 85-86).

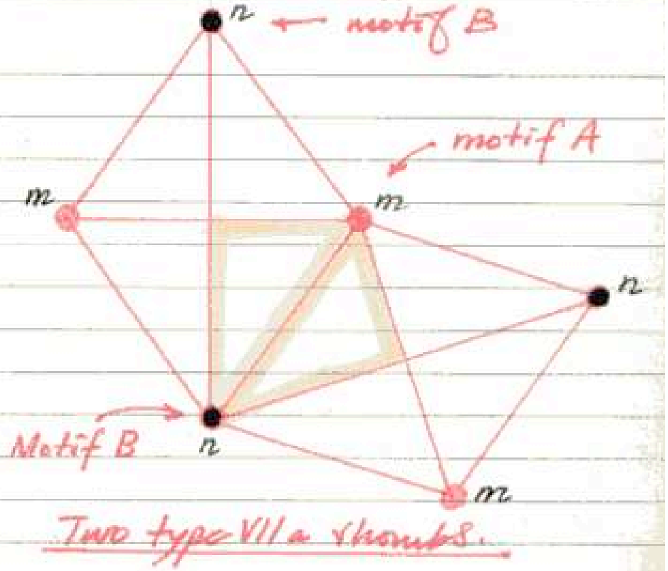
Figs A & B opposite show the method of joining Type VIIa rhombs used in Sp1(3x2)12,8 and H1(3x2)9,12. This cannot of course form a repeating pattern with (3x2)10,10/VIIa rhombs. Method C & D, using both a and b rhombs, can only be used in Rp1′(3x2)10,10/VII (which is in effect a dichromatic colouring of Rp1(3x2)10,10 — see p. 87). Motifs in Type VII should only be distinguished as A and B, as shown opposite.

Wednesday, AUGUST 17, 1966

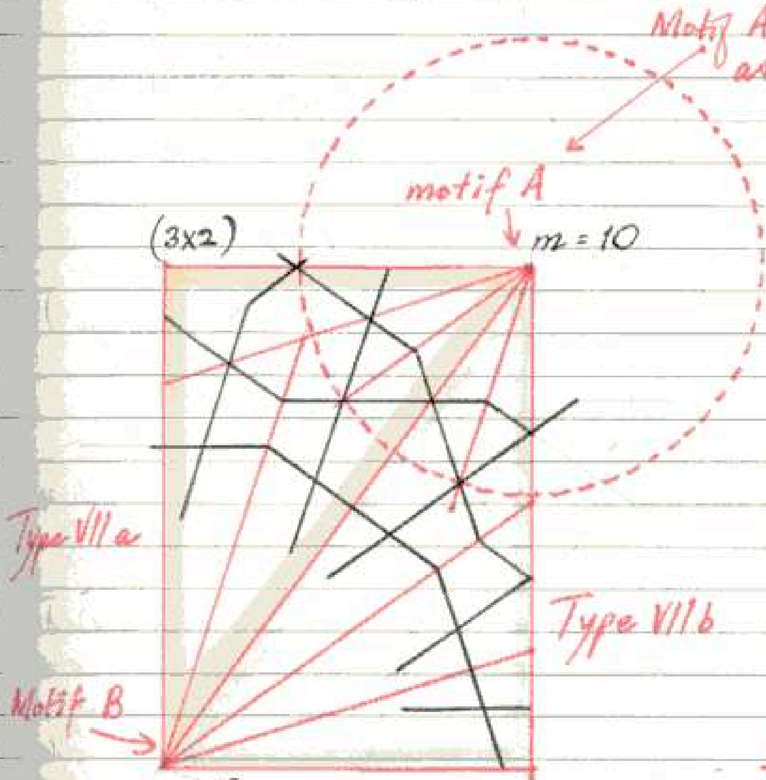
see also pp. 83-84. and pp. 85-86



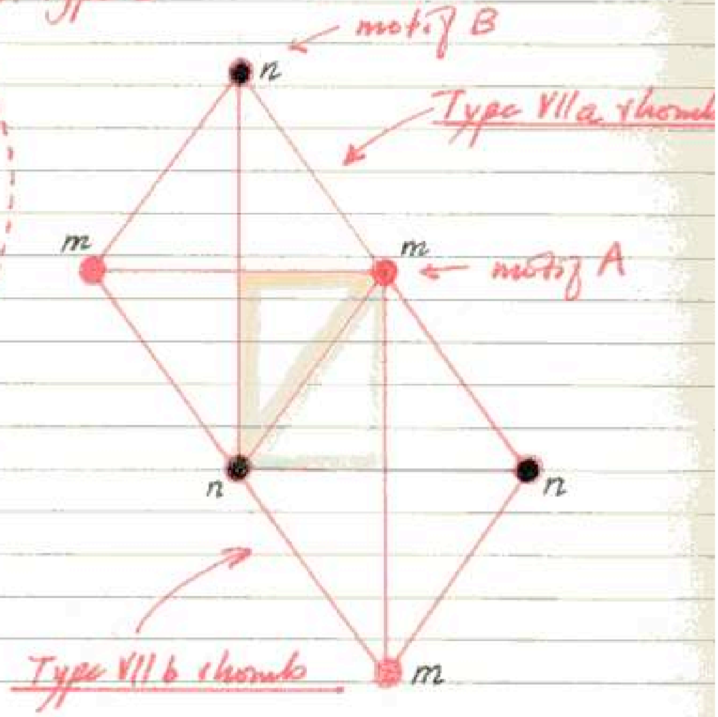
A



B



C



D

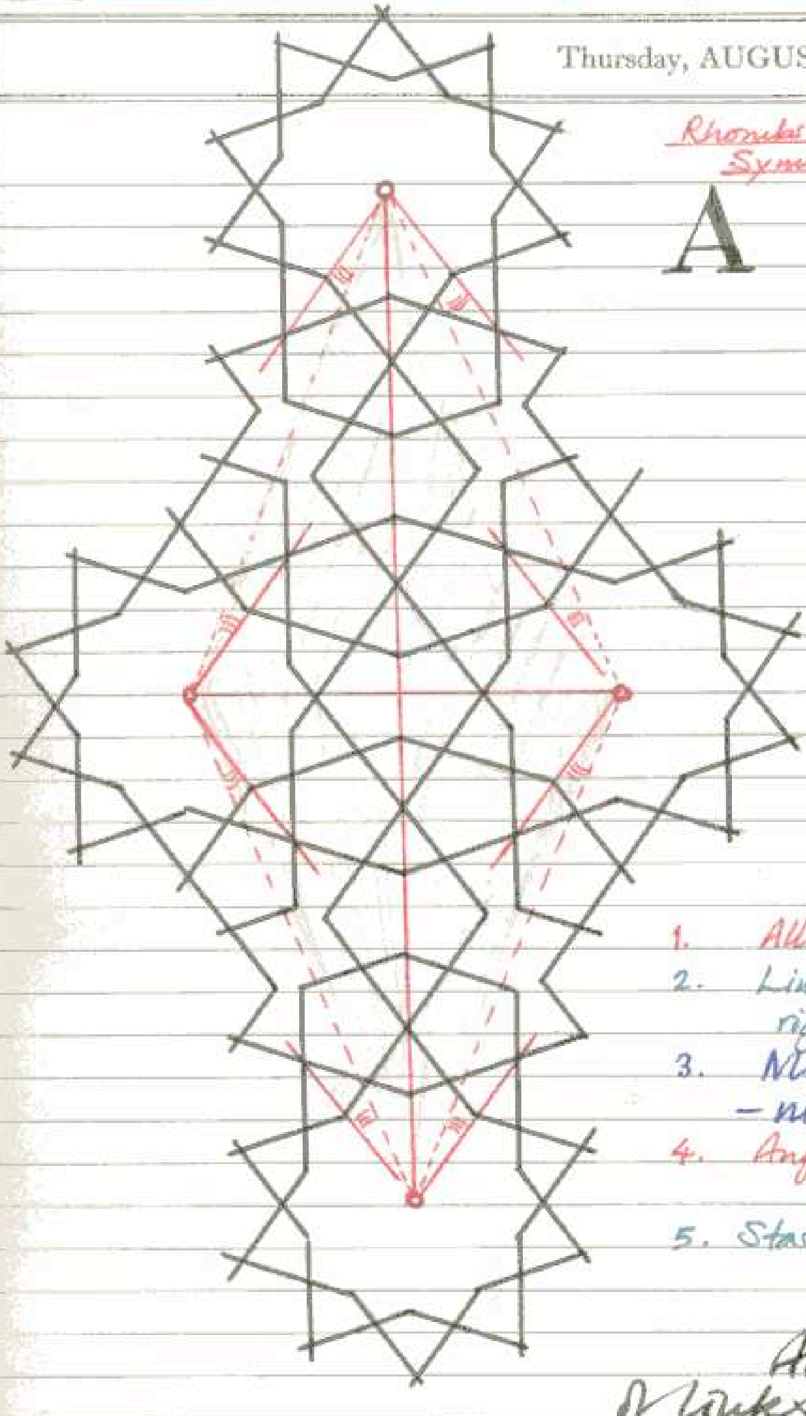
When $m = n = 10$ it is immaterial which motif is m and which n (i.e. types VIIa, VIIb rhombs) but when $m \neq n$ it becomes important to distinguish them.

To what extent must we distinguish between type I motifs & patterns? With regard to motifs, type VII is "mixed": types I and VIII motifs are "pure".

Parallel Links along Edges of Rhombs

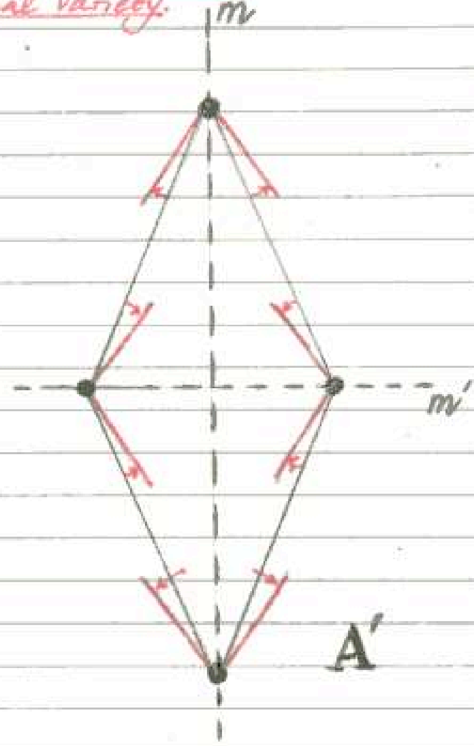
Office
Wed 6 Feb 1985.

Thursday, AUGUST 18, 1966



Rhombs with parallel links;
Symmetrical Variety.

A



A'

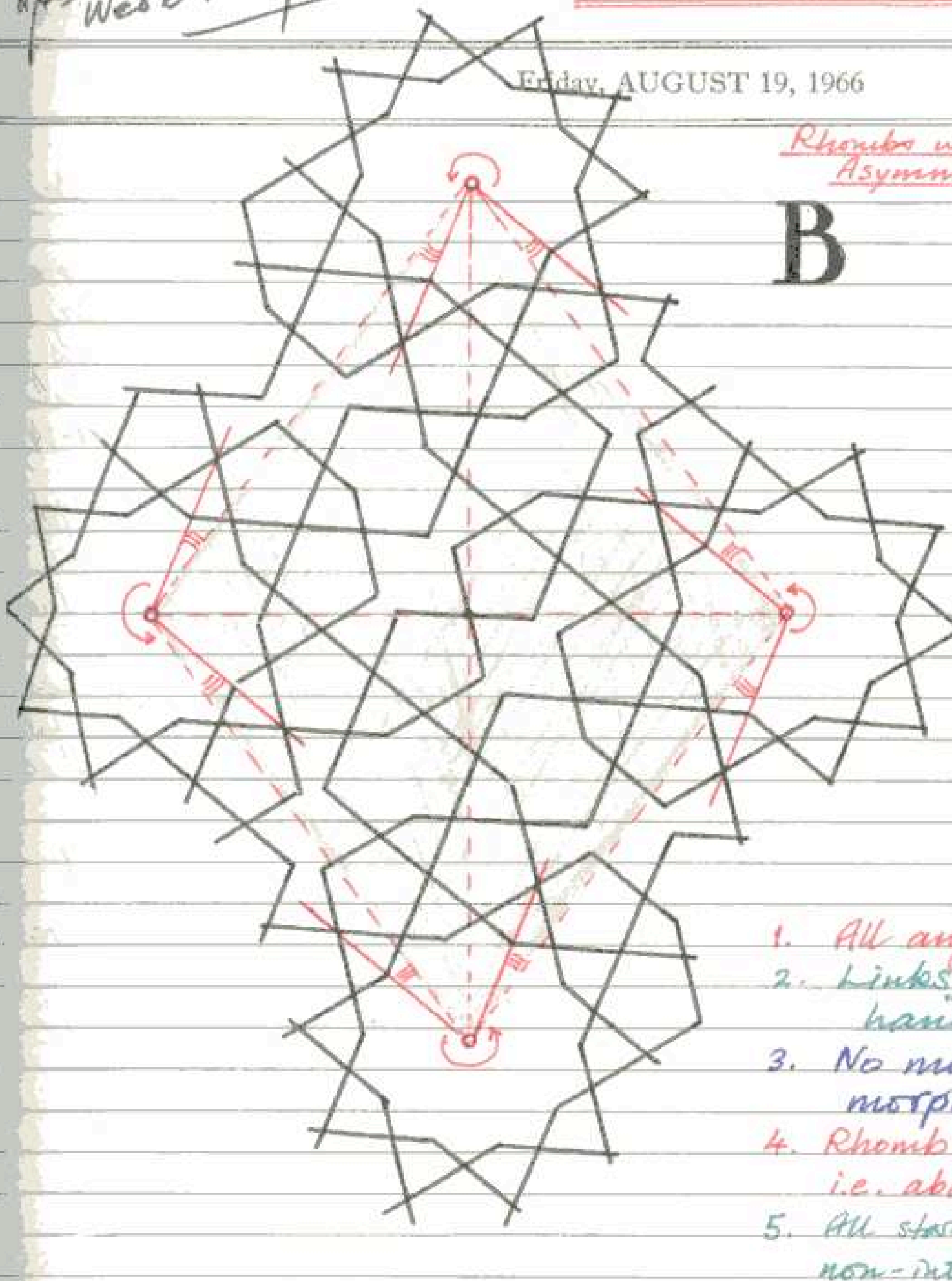
1. All angles of skew are equal.
2. Links are alternately left- and right-handed on sides of rhomb.
3. Mirror axes are present (m, m') – no enantiomorphic varieties.
4. Angles of rhombus usually non-integra
5. Stars not rotated relative to rhomb axes

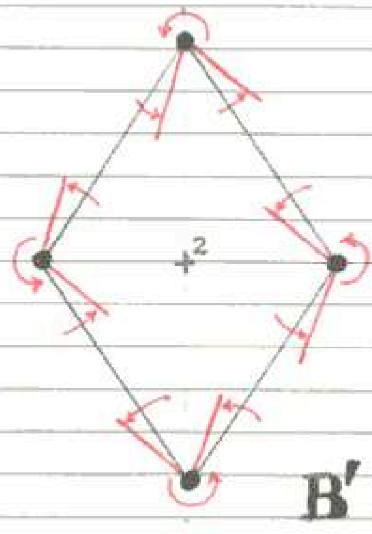
Although the different kinds of links possible between adjacent stars have been defined and illustrated (p. 157 et seq.) no mention has been made in this notebook of their use in rhombs or other fundamental pattern shapes. Rhombic arrangements are indeed possible in which parallel links occur along each rhomb edge. Two major varieties are possible, examples of which are illustrated on pp. 193–194. These may be distinguished as "symmetrical" and "asym-

Friday, AUGUST 19, 1966

Rhombs with parallel links;
Asymmetrical Variety.

B





B′

1. All angles of skew are equal.
2. Links are all of the same handedness.
3. No mirror axes, so enantiomorphic varieties are possible
4. Rhomb retains integral angles, i.e. above rhomb is still [6×4].
5. All stars rotated through a small non-integral angle in the same sense relative to the rhomb.

-metrical"; The symmetrical variety occurs frequently in authentic decagonal patterns, but I know of no authentic examples of the asymmetrical variety. Had any been discovered Their lack of mirror axes would probably have debarred them from use as wall decoration. Repeating patterns are possible with parallel links along the edges of rectangles and parallelograms, but again, no authentic examples seem to occur. The pattern above should be adaptable to (3×2)12,8 and (3×2)9,12, but no drawings are yet available

AfAm Fri 8 Feb 1985

Saturday, AUGUST 20, 1966

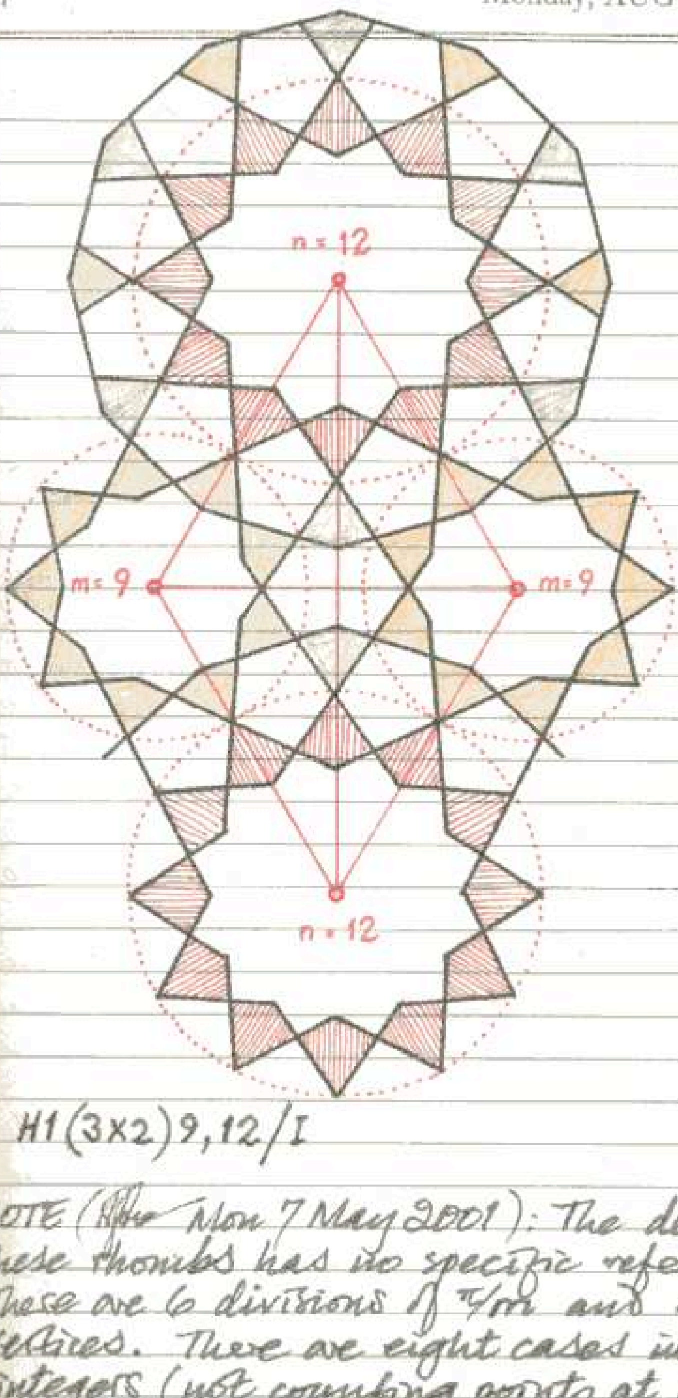
Pattern Types in the [6X4] Rhomb – Non central cases.

The central solution, 10,10, in this series of integral rhombs has received some detailed treatment already in this notebook (pp. 25–66) together with illustrations of the main pattern realizations. The two most useful solutions among the non-central members (see pp 11-14) are [6X4] 9,12 which is suitable to an H1 rhombic tiling, and [6X4] 12,8, which forms the Sp1 tiling of squares. All three cases are by far the most common in authentic Islamic ornament of the [6X4] series of rhombs (the extreme outer members of this series either do not occur in authentic ornament or only rarely so; at least, I have never seen any examples in my searches for patterns and pattern types). A number of pattern types for both [6X4] 9,12 and [6X4] 12,8 have been analysed together with [6X4] 10,10 and briefly illustrated previously but the features of the overall patterns have not been indicated before in this notebook. Pages on which these are mentioned are:—

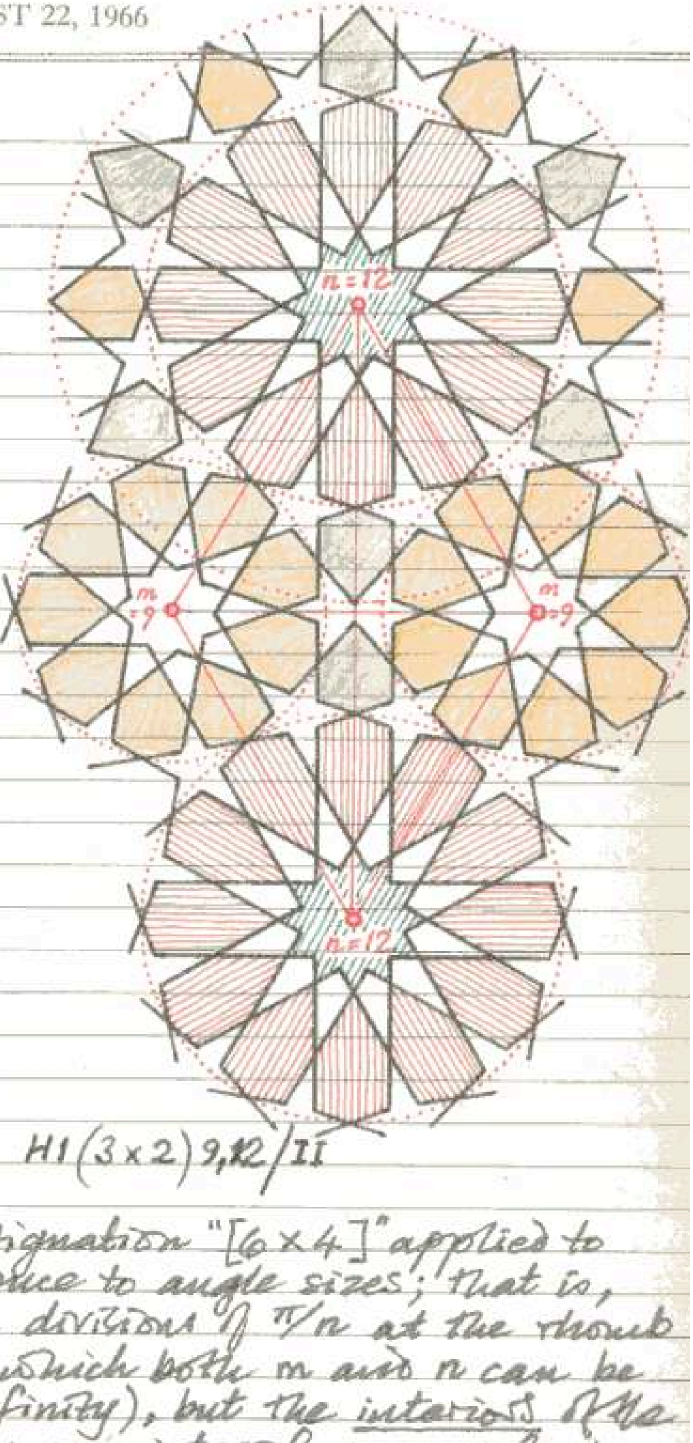
	9,12	10,10		12,8	older type numbers
Type I*	p. 31	31	48	31	I1
II	29,30, 118	29,30	46	29,30, 77	Ia
III			46		Ib
IV		42	56		IX
V		42	56		X
VI	36	36	48		III
VII	34	34	50 191,192	34	IV
VIII	32	32	50	32	V
IX		44	52		VI
X		44	52		VII
XI	38		54		VIII
XII			54		—

Sunday, AUGUST 21, 1966

* These are revised, "blue" designations — see pp. 46–56 — used in my preliminary paper "Islamic Star Patterns".



H1(3×2)9,12/I



H1(3×2)9,12/II

NOTE (Mon 7 May 2001): The designation "[6×4]" applied to these rhombs has no specific reference to angle sizes; that is, these are 6 divisions of π/m and 4 divisions of π/n at the rhomb vertices. There are eight cases in which both m and n can be integers (not counting points at infinity), but the interiors of the rhombs can be constructed for any non-integral m, n values satisfying the relationships $m = 6n/(n-4)$, $n = 4m/(m-6)$ or $3/m + 2/n + 1/2 = 1$.

Wed 13 Feb 1985

Solving Group "A" Patterns in (3 x 2) Rhombs (p. 83)

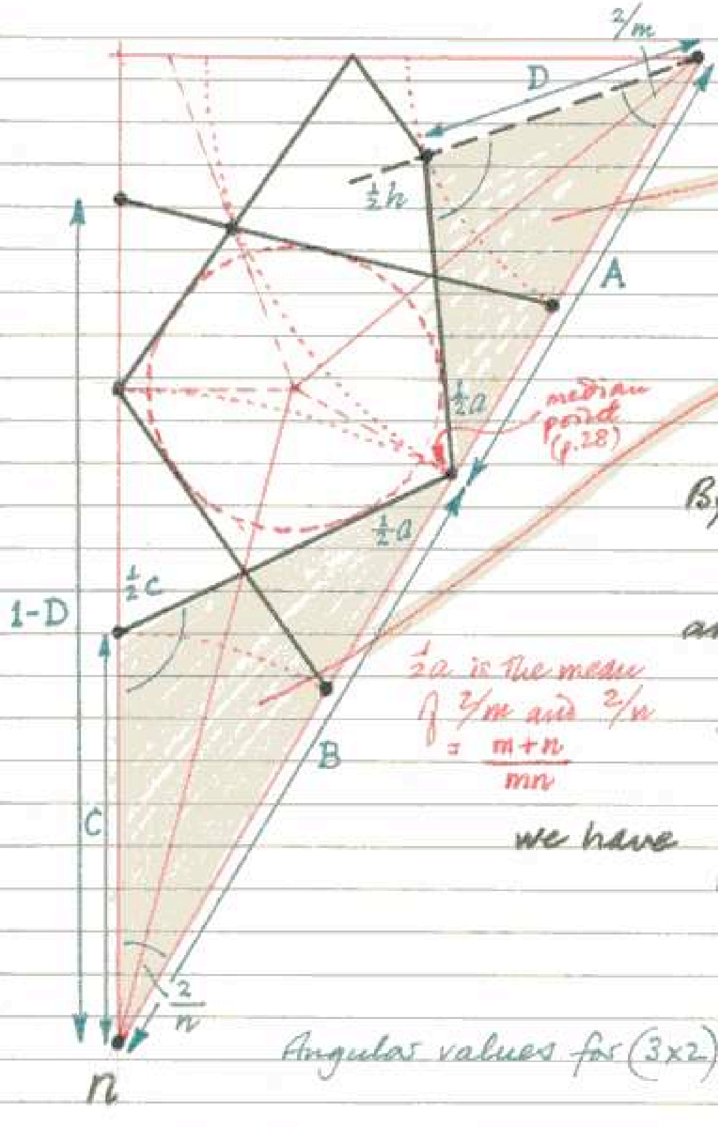
Since the angles at every intersection of a pair of pattern lines can be precisely determined in this group, it is possible to define the lengths of all line segments of need be, and the lengths of the radii at different levels within the constituent stars. This is what we mean when we speak of "solving" a pattern.

The eight patterns forming type I are rigidly determined, so that the angles are "fixed" by the number of points in the stars forming each pattern. Indeed, this is so for every type I pattern, whether or not it belongs to the (3x2) rhomb series. General expressions for the angles of all (pxq) m, n/I rhombs are given on pp. 67-68, and for (3x2) rhombs on pp. 73-74. Actual angular values for each member of the (3x2) m, n/I series are given in the table at the bottom of p. 74.

In type I patterns, from a practical point of view we only need the four quantities A, D, B and C to enable the patterns to be constructed with the minimum of trouble. The figure on p. 198 opposite shows the means for obtaining these, and a table is also given showing the actual values for each member of the (3x2) series. As an additional check on accuracy the radius of the n-gon surrounding the n-star is also given, i.e. 1-D.

Most published constructions obtain the median point (see p. 28) by the first collateral method, but as explained previously the second collateral method is probably slightly more accurate if one is not in possession of accurate values for radii A and B.

SOLUTION OF (3x2) TYPE I RHOMBS



from the sine rule

$$D = \frac{A \sin \frac{1}{2} a}{\sin \frac{1}{2} h} \qquad \ldots\ldots 1$$

and

$$C = \frac{B \sin \frac{1}{2} a}{\sin \frac{1}{2} c} \qquad \ldots\ldots 2$$

By convention,

$$A + B = 1 \qquad \ldots\ldots 3$$

and since

$$\frac{A}{B} = \frac{\cot \pi/m}{\cot \pi/n} \qquad \ldots\ldots 4$$

we have

see also p. 204

$$B = 1 \Big/ \left(\frac{\cot \pi/m}{\cot \pi/n} + 1 \right) \qquad \ldots\ldots 5$$

$$A = 1 - B \qquad \ldots\ldots 6$$

Angular values for (3x2) type I are given on p. 74

		7,28	8,16	9,12	10,10	12,8	14,7	18,6	30,5		
m	A	0.1896	0.32442	0.42402	0.5	0.60721	0.678	0.76604	0.87362	A	n
	D	0.1015	0.18377	0.25179	0.30902	0.40010	0.4695	0.56858	0.72256	D	
n	B	0.8104	0.67558	0.57598	0.5	0.39279	0.322	0.23396	0.12638	B	m
	C	0.6098	0.45141	0.36452	0.30902	0.24118	0.2005	0.15270	0.09257	C	
	1-D	0.8985	0.81623	0.74821	0.69098	0.59990	0.5305	0.43142	0.27744	1-D	

Wed 13 Feb 1985

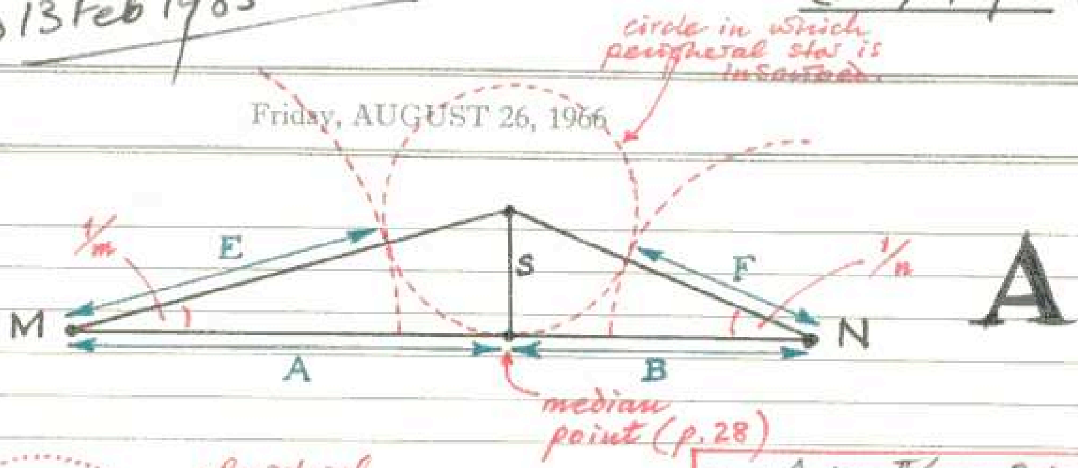
Solution of (3x2) m, n / type II

The solution of type II patterns, i.e. those with tangent "geometrical rosettes", is less straightforward than the solution of type I patterns. This is because whereas type I is completely determined with regard to all angles and lengths of line segments, the exact slopes of pattern lines and therefore sizes of angles in type II patterns can be infinitely varied, even in the case of what we shall call the standard construction. In the standard construction for type II patterns the size of the peripheral circles, with radius s (see figs A, C opposite) determines the radii of the m and n outer stars (E and F respectively). The peripheral circles are of course the circles within which the peripheral stars are inscribed, i.e. all points of the peripheral stars lie on this same circle in the standard construction.

However, the radii of the re-entrant points of the inner stars of each rosette, that is, the inner radii, are still infinitely variable, since the slopes of the sides and terminal segments of the rosettes are also infinitely variable, the outer points of the peripheral stars acting as fixed nodal points. In authentic versions of type II patterns one rosette, usually the smaller, is constructed as parallel-sided with collinear terminal segments*, and this construction is the basis of the calculations given in figs. B & C on p. 200, opposite. This version may be referred to as the principal standard construction for type II patterns. Frequently, however, the larger rosette also is given parallel sides, and if the distance between the parallel sides is identical in both large and small rosettes we may term this construction the secondary standard construction (or "parallel" standard construction). This variety of H1(3x2)9,12/II is illustrated on p. 196. However, this only works well when m<n, since otherwise the congruence of interstitial and m outer cells is destroyed.

* Standard Type II patterns may thus be distinguished as m- or n-parallel (p. 202 footnote)

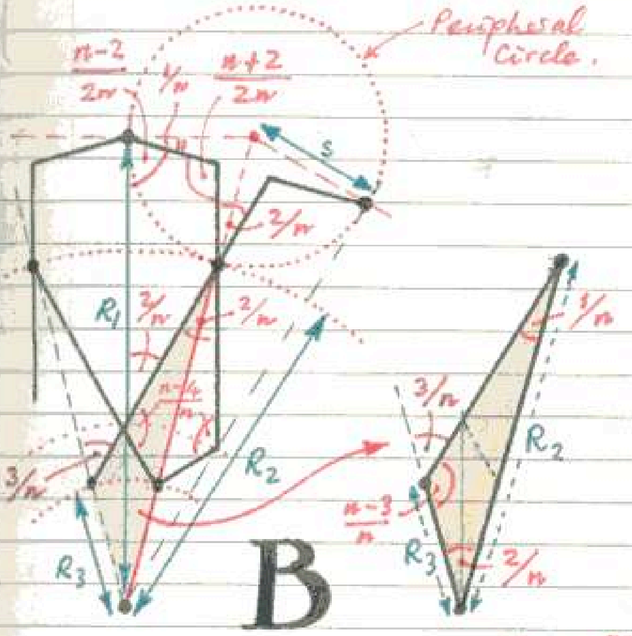
Friday, AUGUST 26, 1966



$\boxed{s = A \tan \pi/m = B \tan \pi/n}$ 1

$$E = \frac{A}{\cos \pi/m} - s \quad \text{........2}$$

$$F = \frac{B}{\cos \pi/n} - s \quad \text{........3}$$

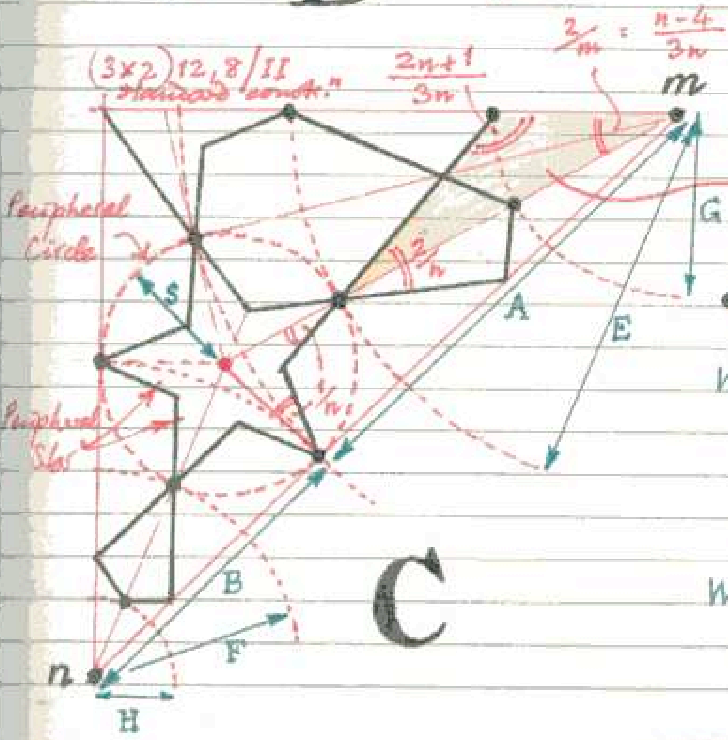


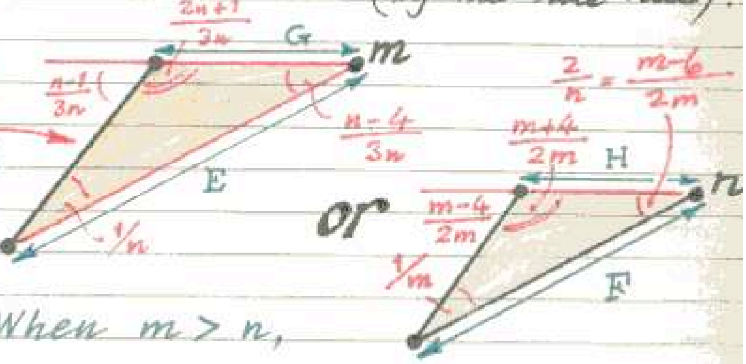
R_1 is either A or B above (usually the smaller)

R_2 is either E or F above (usually the smaller)

$$R_3 = \frac{R_2 \sin \pi/n}{\sin \frac{n-3}{n} \cdot \pi} \quad \text{........4}$$

(by the sine rule).





When $m > n$,

$$G = \frac{E \sin \pi/n}{\sin \frac{2n+1}{3n} \cdot \pi} \quad \text{........5}$$

When $m < n$,

$$H = \frac{F \sin \pi/m}{\sin \frac{m+4}{2m} \cdot \pi} \quad \text{........6}$$